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REGULARIZATION OF THE SOLUTION OF THE CAUCHY PROBLEM FOR A SYSTEM OF EQUATIONS OF ELASTICITY THEORY IN AN INFINITE DOMAIN

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Abstract

The paper discusses continuation of a solution of the elasticity system in a domain from its values and the values of its stresses on part of the boundary, i.e. the Cauchy problem for the Lame system.

Keywords

Cauchy problem, Lame system, Elasticity theory, Infinite domain, Carleman matrix, Carleman function, Regularization, Fundamental solutions, Laplace operator, Harmonic function.

Let R^2 be the real Euclidean space, D be a domain in R^2 with piecewise-smooth boundary ∂D and S be a smooth part of ∂D.

Let $x=(x_1,x_2)$, $y=(y_1,y_2)$ be points of R^2, and let $U(x)$ satisfy in the domain D the homogeneous Lame system:

$$\mathrm{L}\,U(x)=0, \qquad (1)$$

where $\mathrm{L}=\mu\Delta+(\lambda+\mu)\,grad\,div$, Δ is the Laplace operator, λ,μ are the Lame constants such that $\mu \quad 0$, $\lambda \quad -2\mu$.

Set

$$U(y)=f(y),$$
$$T(\partial_y,n)U(y)=g(y), \quad y \quad S \qquad (2)$$

where $f(y)=(f_1(y),f_2(y))$ and $g(y)=(g_1(y),g_2(y))$ are given continuous vector functions on S, $T(\partial_y,n)$ is a differential operator with components

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$T_{ij}(\partial_y, n) = \mu\,\delta_{ij}\dfrac{\partial}{\partial n} + \lambda n_j(y)\dfrac{\partial}{\partial y_j} + \mu n_j(y)\dfrac{\partial}{\partial y_i}, \quad i,j = 1,2$, where δ_{ij} is the Kronecker symbol.

It is required to extend $U(y)$ into D, using the given f and g.

In this paper we present a solution method based on the Carleman function [1].

Using the Carleman function for the Cauchy problem for the Laplace equation, the Carleman matrix for the Cauchy problem for systems of elasticity equations in bounded domains was constructed in [2,3], and for the multidimensional case in special unbounded domains the problem was considered in [2].

Definition 1. A $(2x2)$ matrix $\Pi(y,x,\sigma)$ satisfying the following two conditions is called the Carleman matrix of problem (1), (2):

1) $\Pi(y,x,\sigma) = \Gamma(y,x) + G(y,x,\sigma)$,

where σ is a positive numerical parameter, the matrix $G(y,x,\sigma)$ with respect to the variable y satisfies system (1) everywhere in D, $\Gamma(y,x)$ is the matrix of fundamental solutions of the elasticity system.

2) $\int_{\partial D\setminus S} \left(|\Pi(y,x,\sigma)| + |T(\sigma_y, n)\Pi(y,x,\sigma)|\right) ds_y \quad \varepsilon(\sigma)$,

where $\varepsilon(\sigma) \to 0$, as $\sigma \to$ uniformly in x on compact subsets of D; here and below $|\Pi|$ denotes the Euclidean norm of the matrix $\Pi = \|\Pi_{ij}\|$, i.e. $|\Pi| = \left(\sum_{i,j=1}^{m} \Pi_{ij}^2\right)^{1/2}$, in particular $|U| = \left(\sum_{i=1}^{m} U_i^2\right)^{1/2}$, for the vector $U = (U_1,...,U_m)$.

Definition 2. The vector function $U(y) = (U_1(y),...,U_m(y))$ is called regular in D, if it is continuous together with its second-order partial derivatives in D and first-order partial derivatives on $\overline{D} = D \quad \partial D$.

Theorem 1. Every regular solution $U(x)$ of equation (1) in the domain D is given by the formula [4]

$$U(x) = \int_{\partial D} \left(\Gamma(y,x)T(\partial_y, n)U(y) - U(y)\{T(\partial_y, n)\Gamma(y,x)\}\right) ds_y, x \quad D.$$

It is easy to see that, from the definition of the Carleman matrix and the well-known Green integral formula, in the latter formula the fundamental solution $\Gamma(y,x)$ can be replaced by the Carleman matrix. Hence the following holds.

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Theorem 2. Every regular solution $U(x)$ of equation (1) in the domain D is given by the formula

$$U(x)=\int_{\partial D}\left(\Pi(y,x,\sigma)T(\partial_y,n)U(y)-U(y)\{T(\partial_y,n)\Pi(y,x,\sigma)\}\right)ds_y,\ x\quad D\ , \tag{3}$$

where $\Pi(y,x,\sigma)$ is the Carleman matrix.

Let $K(\omega),\ \omega=u+iv$, (u,v real) be an entire function taking real values on the real axis and satisfying the conditions.

$$K(u)\quad 0,\ \sup_{V\ 1'}\left|v^p K^{(p)}(\omega)\right|=M(p,u)<\quad ,\quad p=0,1,2\ ,\ u\quad R^1$$

Set $\alpha=|y_1-x_1|$.

Define the function $\phi(y,x)$ for $\alpha>0$ by the following relation:

$$-2\pi\ K(x_2)\phi\ (y,x)=\int_0 \mathrm{Im}\frac{K\left(i\sqrt{u^2+\alpha^2}+y_2\right)}{i\sqrt{u^2+\alpha^2}+y_2-v_2}\ \frac{udu}{\sqrt{u^2+\alpha^2}}\ , \tag{4}$$

Using the function $\phi(y,x)$, construct the matrix

$$\Pi(y,x)=\left\|\Pi_{ij}(y,x)\right\|_{2x2}=\left\|\frac{\lambda+3\mu}{2\mu(\lambda+2\mu)}\delta_{ij}\phi(y,x)-\frac{(\lambda+\mu)}{2\mu(\lambda+2\mu)}(y_j-x_j)\frac{\partial}{\partial y_i}\phi\ (y,x)\right\|_{2x2} \tag{5}$$

Let now D be an infinite domain in R^2, lying inside the strip of smallest width defined by the inequality $0<y_2<h,\ h=\frac{\pi}{\rho}\ ,\rho>0$. Moreover, ∂D consists of the smooth part S and $y_2=0$. We shall assume that for some $b_0>0$ the area of ∂D satisfies the growth condition

$$\int_{\partial D}\exp\left(-b_0\exp\rho_0|y_1|\right)ds_y<\quad ,\ 0<\rho_0<\rho, \tag{6}$$

For $\sigma\quad 0$ in (4)-(6) let

$$K(\omega)=\exp\ \sigma\omega-bchi\rho_1\ \omega-\frac{h}{2}\ -b_1 chi\rho_0\ \omega-\frac{h}{2}\qquad ,$$

where $\omega=y_2+iv,\quad \rho_0,\rho_1\ (0,\rho),\ 0<x_2<h,\ b>0\quad b_1>b_0\ \cos\rho_0\frac{h}{2}\ ^{-1}$.

Take $\phi(y,x,\sigma)=\phi\ (y,x)$ and $\Pi(y,x,\sigma)=\Pi(y,x)$. In [1] the following is proved.

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Lemma 1. The function $\phi(y,x,\sigma)$ can be represented in the form

$$\phi(y,x)=\frac{1}{2\pi}\ln\frac{1}{r}+g(y,x,\sigma),\qquad r=|y-x|$$

where $g(y,x,\sigma),$ is a function defined for all values of y,x and harmonic with respect to the variable y throughout R^2.

From this lemma it follows that the matrix $\Pi(y,x,\sigma)$ is the Carleman matrix of problem (1), (2).

Assume that $U(x)$ $A(D)$ and

$$|U(y)|+|T(\partial y,n)U(y)| \quad M,\ y \quad \partial D. \qquad (7)$$

Under these assumptions, formula (7) holds, where

$$K(\omega)=(\omega-x_2+2h)^{-1}\exp\sigma\omega\ ,\ \omega=i\sqrt{u^2+\alpha^2}+y_2,$$

Introduce the notation

$$U_\sigma(x)=\int_S[\Pi(y,x,\sigma)\{T(\partial y,n)U(y)\}-U(y)\{T(\partial y,n)\Pi(y,x,\sigma)\}]ds_y,\ x \quad D.$$

$$A_\rho(D)=\{U(x) \quad A(D):|U(y)|+|T(\partial y,n)U(y)| \quad \exp(0(\exp|y_1|)),\ y_1\to \ ,\ y \quad D\}.$$

Theorem 3. Let $U(x)$ $A_\rho(D)$ satisfy boundary condition (7). Then

$$|U(x)-U_\sigma(x)| \quad MC(\rho,x)\sigma\exp(-\sigma x_2),\ x \quad D,$$

where $C(\rho,x)=C(\rho)\int_{y_2=0}\frac{ds_y}{r^3}$

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