ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23
American Academic publishers, volume 6, issue 03,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai

THE INTERSECTION OF POLYHEDRA WITH A PLANE.

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Abstract: This article analyzes the spatial-geometric properties of polyhedra and the theoretical foundations of their intersection with a plane. The definition of a polyhedron, its main elements (faces, edges, vertices), and its types are scientifically explained using examples of prisms, pyramids, and regular polyhedra. The property that the cross-section formed by intersecting a convex polyhedron with a plane is itself a convex polygon is substantiated through geometric and analytical approaches.

Keywords: Polyhedron, plane, cross-section, convex polyhedron, spatial geometry, stereometry, prism, pyramid, regular polyhedron, analytic geometry, plane equation, intersection point, geometric modeling.

Polyhedra are bodies in space bounded by a finite number of plane polygons. In general cases, polyhedra can be composed of prisms, pyramids, or other complex shapes. The faces of a polyhedron's surface are these polygons, the edges are the line segments connecting their vertices, and the vertices are the points where the edges meet.

The figure formed when a polyhedron is intersected by a plane is called a cross-section. From a geometric point of view, this cross-section is the set of all points common to the plane and the surface of the polyhedron, and it also takes the form of a polygon.

Definition of a Plane: A plane in R3 space is a two-dimensional, infinite flat surface, expressed by the general algebraic equation with 3 spatial variables:

ax+by+cz+d=0ax + by + cz + d = 0ax+by+cz+d=0

Here, a, b, c represent the normal to the plane, and d is a parameter related to its distance from the origin.

Polyhedron Theory: Polyhedra, as geometric objects, are often convex, meaning all their interior dihedral angles are ≤ 180°. The cross-section resulting from the intersection of a convex polyhedron with any plane is also a convex polygon.

Principle of Intersecting a Polyhedron with a Plane

The cross-section of a polyhedron with a plane is constructed through the following process:

1. Find the points where each edge of the polyhedron intersects the plane.
2. Connect these intersection points in sequential order.
3. Identify the resulting closed polygon as the cross-section.

For example, when a cube is intersected by a plane, the shape of the resulting cross-section depends on the orientation of the plane and can range from a triangle to a hexagon.

Mathematical Considerations:

Theorem: If P is a convex polyhedron and L is a plane that intersects it, then the intersection P∩L forms a closed, convex polygon.

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Proof (intuitive): The intersection of each edge of a polyhedron with a plane is a line or a point. Convexity ensures that when these points are connected in a linear order, the figure formed along the line stretched between them will also be convex. This is because, even though the plane is infinite, the cross-section remains within the boundary of the polyhedron.

Practical solution methods: There are two common approaches to drawing the cross-sections of a polyhedron:

1. **Edge intersection method: Finding the intersection point of each edge with the plane by solving the plane's equation, and then connecting these points along the faces.**
2. **Analytical intersection of faces: Drawing the projection of a face onto the plane and working with the elements of the resulting cross-sectional figure.**

In conclusion, polyhedra and the planar sections made with them form the basis of elementary spatial geometry. If we consider this based on mathematical theory, convexity properties, and drawing methods:

- The cross-section of a convex polyhedron is a convex polygon.
- The type of resulting cross-section depends on the orientation of the plane relative to the faces.
- Cross-sections defined by lines and points can be rendered into precise images.

Possible shapes of the cross-section: If a polyhedron has 'n' faces, the number of sides of the cross-sectional polygon will not exceed 'n' at most.

For example, when a cube is intersected by a plane:

- if it intersects 3 edges, a triangle is formed,
- if it intersects 4 edges, a quadrilateral is formed,
- if it intersects 5 edges, a pentagon is formed,
- if it intersects 6 edges, a hexagon is formed.

This result is substantiated through Euler's formula and combinatorial analysis related to the number of edges.

Methods for determining cross-sections:

Geometric (graphical) method

- Finding the intersection points of the edges with the plane;
- Projecting them onto the drawing plane;
- Connecting them in order.

This method is used in engineering drawing.

Analytical method

- Expressing each edge in parametric form;
- Substituting into the plane equation;
- Solving the system of linear equations.

This method is widely used in computer graphics and 3D modeling.

List of references:

1. Yodgorov, J. *Chizma geometriya* [Descriptive Geometry]. - Tashkent: O'qituvchi, 2018. - 232 pages. (Chapter on polyhedra and their intersection with a plane, pp. 100-120).
2. Solihov, S.S., & Karimov, A.A. "Muntazam ko'pyoqlar kesimlari" [Cross-sections of Regular Polyhedra]. - *IJODKOR O'QITUVCHI*, 2025, Vol. 4, No. 47. - pp. 15-22 (abstract and introduction on pp. 15-16).
3. **"Cross section (geometry)." - Wikipedia, "Cross section of polyhedron" section (accessed: 2026).**

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4. "Ko'pyoqlarning tekislik bilan kesilishi va uning yoyilmasini yasash" [The Intersection of Polyhedra with a Plane and the Construction of its Net]. - Term paper, 2024. - pp. 10-18 (algorithmic construction methods).

5. Khojamov, Z. S. (2021). "Problems of modeling on the subject 'drawing geometry and engineering graphics'." *World Bulletin of Social Sciences*, 5, 90-91.

6. Khujamov, Z. S., & Saparov, Kh. R. (2023). "Formation of Graphic Literacy of Requirements in Drawing Lessons." *Journal of Intellectual Property and Human Rights*, 2 (3), 24-26.

7. Khojamov, Z. S. (2021). Models of Computer Graphics of Special Curved Lines Lying on a Curved Surface. European Journal of Agricultural and Rural Education, 2 (12), 85-88.