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LINEAR ALGEBRA AND ITS PRACTICAL APPLICATIONS IN ECONOMIC PROBLEMS

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Abstract. Linear algebra is one of the fundamental mathematical tools widely applied across various fields of economics. This paper examines several practical applications of linear algebra in solving economic problems. In particular, it highlights the possibilities of analyzing data and determining optimal decisions through the modeling of economic processes using linear algebraic methods. The roles of key concepts - such as matrices, vectors, systems of linear equations, and determinants - in economic modeling and analysis are discussed. The results of the study substantiate the practical significance of linear algebra in economics and demonstrate its effectiveness in addressing complex economic problems.

Keywords: linear algebra, matrices, determinants, Leontief model, economic modeling, technological coefficients.

The effective management of a multi-sector economy requires maintaining equilibrium among its sectors. Each sector simultaneously acts as both a producer and a consumer of goods produced by other sectors. Therefore, the quantitative representation and analysis of the interdependencies arising from the production and consumption of various goods across sectors constitute a complex problem.

Systems of linear equations, which are a fundamental component of linear algebra, are widely used in solving problems related to economic planning and management. In particular, they provide a framework for constructing and analyzing mathematical models that describe intersectoral relationships. In what follows, the mathematical model of intersectoral balance is considered.

Quantitative analysis of the economy, especially the study of the social production process, leads to the examination of interconnected flows of goods and services. From this perspective, the economic system can be viewed as a set of sectors, each specializing in the production of a specific type of product. Goods produced by enterprises are exchanged, resulting in the formation of product flows between sectors. The emergence of such flows is inevitable, as each sector utilizes or consumes the outputs of other sectors in its own production process.

For the normal functioning of the economy, one of the essential conditions is the existence of a balance between production costs and the total gross output across all sectors. In this context, it is necessary to take into account that a portion of the produced output does not return to the production sphere, but is instead allocated to personal consumption, accumulation, or export.

Assuming that the gross output of the economic system is produced across nnn interrelated sectors, we consider a time period that encompasses a complete production cycle.

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Let $x_1, x_2, \ldots, x_n$ denote the gross output levels (in natural units) produced by the first, second, …, n – th sectors, respectively. Suppose that, in the period under consideration, x_1 represents the amount of metal produced by the metallurgical sector, x_2 denotes the output of the chemical industry, and x_3 corresponds to the number of passenger cars produced by the automotive sector, and so on.

The vector $x_1, x_2, \ldots, x_n$ is called the gross output vector of the system.

Let x_{ik} denote the amount of output from the i – th sector required to produce one unit of output in the k – th sector. For example, in our case, x_{13} represents the amount of output from the first sector (i.e., metal) required to produce x_3 units of automobiles.

Let y_i denote the amount of final output of the i – th sector that does not return to the production process. Then the vector $y(y_1, y_2, \ldots, y_n)$ is called the final demand (final output) vector of the system [1].

The material balance scheme for the product x_{ik} of the i – th sector $(i = \overline{1,\ldots,n})$ of the system can be represented, according to the principle of "production and distribution of output," in the following form (Table 1).

Table 1. Production and distribution of output

Intermediate consumption (production use)	Final output	Groos output
$x_{11}, x_{12}, \ldots, x_{1n}$	y_1	x_1
$x_{21}, x_{22}, \ldots, x_{2n}$	y_2	x_2
……………	…	…
$x_{n1}, x_{n2}, \ldots, x_{nn}$	y_n	x_n

The flow balance equations of the material balance are expressed in the form

$$x_i = \sum_{k=1}^{n} x_{k1} + y_i, \quad i = 1, 2, \ldots, n$$

The above expression can be represented in the form of a table (Table 2).

Table 2. Product balance table

k / i	1	2	…	n	x

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1	x_{11}	x_{12}	…	x_{1n}	$\sum_{k=1}^{n} x_{1k}$
2	x_{21}	x_{22}	…	x_{2n}	$\sum_{k=1}^{n} x_{2k}$
…	…	…	…	…	…
n	x_{n1}	x_{n2}	…	x_{nn}	$\sum_{k=1}^{n} x_{nk}$
Groos output	x_1	x_2		x_n	
Final output	y_1	y_2		y_n	

Let a_{ik} denote the amount of direct input required to produce one (conventional) unit of output in the $k-$ th sector. The quantities a_{ik} are called direct input coefficients or technological coefficients [4].

It is evident that the total amount of product i consumed by the $k-$th sector, denoted by x_{ik}, is equal to the product of the direct input coefficient a_{ik} and the total output x_k of that sector. That is,

$x_{ik} = a_{ik} x_k$, which reflects the linearity assumption in production costs.

Accordingly, the system of equations can be written in the form

$$x_i = \sum_{k=1}^{n} a_{ik} x_k + y_i, \quad \left(i = \overline{1,\dots,n}\right).$$

This system, in turn, can be expressed in vector–matrix form as

$$X - AX = Y \text{ yoki } \left(E - A\right)X = Y . \qquad (1)$$

Here, $X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$ is the gross output vector, $E_{n\ n} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$ is the identity matrix of order n, $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$ is the matrix of direct input coefficients (also

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called the technological matrix), and $Y = \begin{matrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{matrix}$ is called the final demand (final consumption) vector.

Equation (1) is referred to as the Leontief linear model.

The principal objective of the Leontief model is to determine the gross output vector X given the technological matrix A and the final demand vector Y. The vector X can be obtained using the formula

$$X = (E - A)^{-1} Y = SY \qquad (2)$$

where $S = (E - A)^{-1}$ is called the Leontief inverse or the total requirements matrix.

If, for every nonnegative final demand vector $Y \quad 0$, the system (1) admits a solution $X \quad 0$, then the matrix $A \quad 0$ is said to be productive (or feasible) [2].

Example. Consider the following table:

Sector/ Consumption	**Electric Energy**	**Metallurgy**	**Final Output**	**Gross Output**
Energy	14	21	82	140
Mechanical Engineering	28	12	132	150

Using the data in the table, determine the required gross output levels for each sector, assuming that the output of the energy sector is increased while the mechanical engineering sector remains unchanged.

Solution. Here,

$$x_1 = 120,\ x_2 = 160,\ x_{11} = 14,\ x_{12} = 21,\ x_{21} = 28,\ x_{22} = 15,\ y_1 = 120,\ y_2 = 160.$$

In this case, according to the relation $a_{ij} = \frac{x_{ij}}{x_j} \ (i, j = \overline{1,2})$, we obtain:

$$a_{11} = \frac{x_{11}}{x_1} = \frac{14}{140} = 0{,}1, \quad a_{12} = \frac{x_{12}}{x_2} = \frac{21}{150} = 0{,}14, \quad a_{21} = \frac{x_{21}}{x_1} = \frac{28}{140} = 0{,}2,$$
$$a_{22} = \frac{x_{22}}{x_2} = \frac{12}{150} = 0{,}08.$$

Hence, the matrix of direct input coefficients is

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$$A = \begin{pmatrix} 0,01 & 0,14 \\ 0,2 & 0,08 \end{pmatrix}.$$

Next, we determine the total requirements matrix $(E-A)^{-1}$, which is the inverse of the matrix $E-A$. First, compute:

$$E-A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0,1 & 0,14 \\ 0,2 & 0,08 \end{pmatrix} = \begin{pmatrix} 0,9 & -0,14 \\ -0,2 & 0,92 \end{pmatrix}.$$

Using the formula for the inverse of a matrix, we obtain:

$$(E-A)^{-1} = \frac{1}{0,8}\begin{pmatrix} 0,92 & 0,14 \\ 0,2 & 0,9 \end{pmatrix}.$$

According to the table, the final demand vector is $Y = \begin{pmatrix} 140 \\ 150 \end{pmatrix}.$

Using formula (2), we obtain:

$$X = (E-A)^{-1}\,Y = \frac{1}{0,8}\begin{pmatrix} 0,92 & 0,14 \\ 0,2 & 0,9 \end{pmatrix}\begin{pmatrix} 140 \\ 150 \end{pmatrix} = \frac{1}{0,8}\begin{pmatrix} 0,92\ 140 + 0,14\ 150 \\ 0,2\ 140 + 0,9\ 150 \end{pmatrix} = \begin{pmatrix} 187,25 \\ 203,75 \end{pmatrix}.$$

Thus, the gross output in the energy sector should be increased to 187.25 units, and in the mechanical engineering sector to 203.75 units [3].

Conclusion. The practical significance of linear algebra in economics lies in its ability to provide broader analytical capabilities. It enhances the efficiency of solving complex economic problems, facilitates the optimal allocation of resources, and serves as a fundamental tool in strategic planning. Therefore, studying linear algebra and applying its methods in economics is an essential competence for economists and specialists.

In conclusion, linear algebra proves to be a fundamental mathematical tool in the development of the economy, demonstrating both its relevance and importance. This study contributes to a deeper understanding of the role of linear algebra in economics and promotes its effective application.

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