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APPLICATION OF DIFFERENTIAL CALCULUS IN ECONOMICS

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Abstract. This article analyzes the theoretical and practical aspects of applying differential calculus to solving economic problems. In particular, the economic meaning of derivatives, marginal indicators (cost, revenue, profit), optimization problems, and the concept of demand elasticity are extensively discussed. In addition, the possibilities of modeling economic processes using differential equations are examined. The presented examples demonstrate the effectiveness of differential calculus methods in economic analysis.
Keywords: differential calculus, derivative, marginal cost, marginal revenue, optimization, elasticity, economic model.

Introduction. Modern economics consists of complex, multifactor, and dynamic systems, and mathematical modeling plays a crucial role in their analysis. In particular, differential calculus is one of the main tools for determining the rate of change of economic indicators, making optimal decisions, and ensuring the efficient use of resources.

In economic theory, differential calculus forms the mathematical foundation of marginal analysis. This approach allows for a deeper understanding of the decision-making mechanisms of economic agents and increases the possibility of achieving practical results.

In economics, many processes are expressed through functional relationships. For example, production costs may depend on the volume of output:

$$C=C\left(x\right)$$

Here, x represents the quantity of output produced, and $C\left(x\right)$ denotes the total cost.

When the output volume increases by Δx, the change in costs is given by:

$$\Delta C=C\left(x+\Delta x\right)-C\left(x\right)$$

It is known that the average rate of change of this quantity with respect to one unit of output is calculated as: $\dfrac{\Delta C}{\Delta x}$ [1].

At a more advanced stage of economic analysis, the main focus is on the costs associated with producing an additional unit of output, namely, the marginal cost. To determine this quantity, we consider the ratio of the change in the cost function under infinitesimally small changes. Mathematically, this is expressed through a limiting process, which leads to the concept of the derivative.

In the limit, the derivative is obtained, and it represents the marginal cost:

$$C\ \left(x\right)=\lim_{\Delta x\to 0}\frac{\Delta C}{\Delta x}\ . \qquad (1)$$

The above expression shows that the limiting value of the average rate of change is equal to the derivative of the function, and in economic terms, this derivative represents the marginal cost.

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Thus, from an economic point of view, the derivative represents the change in cost corresponding to a very small change in the volume of production [2].

Example. Suppose the firm's total cost function is given by:

$$y = 100x - \frac{1}{30}x^3.$$

Here, x denotes the quantity of output, and y represents the total production cost.

Find the marginal cost when the production volume is $x = 5$ and $x = 10$.

Solution. First, we find the derivative of the given function:

$$y = 100x - \frac{1}{30}x^3 = 100 - \frac{1}{10}x^2.$$

Now, we calculate the values at the given points:

$$y(5) = 100 - \frac{1}{10}\ 5^2 = 97{,}5\ ,\quad y(10) = 100 - \frac{1}{10}\ 10^2 = 90.$$

The economic interpretation of these results is as follows: when the production volume is 5 units, the cost of producing one additional unit (marginal cost) is 97.5; when the production volume is 10 units, it is 90 [3].

This indicates that as production volume increases, the marginal cost of producing additional units decreases. This situation is explained in economics by economies of scale or increasing production efficiency.

Using derivatives, it is also possible to determine the increment of a dependent variable (function) corresponding to an increment of an independent variable (argument). However, in economic problems, relative changes - i.e., percentage changes - are often more important.

For this reason, the concept of elasticity of a function is introduced. This concept expresses the relative change of a function with respect to the relative change of its argument.

For this reason, the concept of **elasticity of a function** is introduced. This concept expresses the relative change of a function with respect to the relative change of its argument.

The ratios $\frac{\Delta x}{x}, \frac{\Delta y}{y}$ are called the relative increments of the argument and the function, respectively. We consider the ratio of the relative increment of the function to the relative increment of the argument: $\frac{\Delta y}{y} : \frac{\Delta x}{x}$.

Simplifying this expression, we obtain:

$$\frac{\Delta y}{y} : \frac{\Delta x}{x} = \frac{\Delta y}{\Delta x}\ \frac{x}{y}\ .$$

If the function $y = f(x)$ is differentiable, then by passing to the limit we obtain the exact expression for elasticity:

$$E = \lim_{\Delta x \to 0} \frac{\Delta y}{y} : \frac{\Delta x}{x} = \lim_{\Delta x \to 0} \frac{x}{y}\ \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}\ \frac{x}{y} = \frac{x}{y}\frac{dy}{dx}$$

Elasticity is a very important indicator in economic processes, as it determines the sensitivity of one variable with respect to another [4]. In particular:

- If $|E| > 1$, the relationship is **elastic**;

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- If $|E| < 1$, the relationship is **inelastic**;
- If $|E| = 1$, it is **unit elasticity**.

For example, demand elasticity shows how demand responds to changes in price and plays an important role in pricing policy.

Example. Find the elasticity of the function:

$y = 1 + 6x^2 - 4x^3$.

Solution. According to formula (3):

$$E_x(y) = \frac{x}{1+6x^2-4x^3}(12x - 12x^2) = \frac{12x^2 - 12x^3}{1+6x^2-4x^3}.$$

For instance, when $x = 1$, we get:

$$\frac{(12-12)}{7} = 0.$$

This means that when the argument increases by 1% (from 1 to 1.01), the value of the function practically does not change.

Now let us recall some rules used in calculating the elasticity of a function.

Theorem 1. The elasticity of the product of two functions is equal to the sum of their elasticities.

Proof. Based on the derivative formula for the product of two functions:

$$E_x(u \ \ v) = \frac{x}{uv}(u \ \ v) \ = \frac{x}{uv}(u \ v + uv \ ) = \frac{x}{u}u \ + \frac{x}{v}v \ = E_x(u) + E_x(v)$$ or equivalently,

$$E_x(u \ \ v) = E_x(u) + E_x(v).$$

This proves the theorem [5].

Example. Find the elasticity of the function: $y = x^2 \ \ e^{2x}$.

Solution. Let $u = x^2$, $v = e^{2x}$. Then, using formula (4):

$$E_x(y) = \frac{x}{x^2}(x^2) \ + \frac{x}{e^{2x}}(e^{2x}) \ = 2 + 2x,\ E_x = (y) = 2(1+x).$$

Thus,

$$E_x(y) = \frac{x}{x^2}(x^2) \ + \frac{x}{e^{2x}}(e^{2x}) \ = 2 + 2x,\ E_x = (y) = 2(1+x).$$

Theorem 2. The elasticity of the ratio of two functions is equal to the difference of their elasticities:

$$E_x \ \ \frac{u}{v} \ \ = E_x(u) - E_x(v).$$

Example. Find the elasticity of the function: $y = \dfrac{x^3+5}{e^{3x}}$.

Solution. Let $u = x^3 + 5$, $v = e^{3x}$. Then:

$$E_x(y) = E_x(x^3+5) - E_x(e^{3x}) = \frac{x}{x^3+5} \ \ 3x^2 - \frac{x}{e^{3x}} \ \ 3e^{3x} = \frac{3x^3}{x^3+5} - 3x.$$

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So, $E_x(y)=\dfrac{3x^3}{x^3+5}-3x$.

Conclusion. Differential calculus is a powerful mathematical tool in solving economic problems. It enables a deep analysis of economic processes, supports optimal decision-making, and ensures efficient use of resources. Concepts such as marginal analysis, optimization, and elasticity form the foundation of modern economic theory.

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