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APPLICATION OF THE FOURIER SERIES

Department of Mathematics,
University of Economics and Pedagogy
Assistant:
Jamalov Madamin Habibulla ugli
jamolovmadamin7@gmail.com
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Abstract: This article discusses the fundamental concepts of Fourier series theory, its mathematical formulation, and practical applications. It is shown that periodic functions can be represented as a sum of sine and cosine functions. The paper analyzes the methods for determining Fourier coefficients, the convergence conditions of the series, and its applications in physics and engineering. The results highlight the significant role of Fourier series in mathematical analysis, signal processing, and the study of oscillatory processes.

Keywords: Fourier series, periodic function, sine and cosine, Fourier coefficients, convergence, mathematical analysis, signal processing.

Enter. Mathematical analysis, which is one of the important branches of mathematics, allows modeling of real processes and their in-depth analysis. In particular, many phenomena occurring in nature and technology have a periodic nature, and their study is of great importance in the development of science. Many physical phenomena such as vibrations, waves, electric and magnetic fields, sound signals, heat processes are subject to periodic laws. Effective mathematical methods are needed to mathematically represent such processes, to determine their properties, and to apply them to practical problems. One such universal and powerful method is Fourier series theory. The theory of Fourier series was developed by the French scientist Jean-Baptiste Joseph Fourier at the beginning of the 19th century during the research of heat conduction processes. In his researches, Fourier showed that complex functions can be expressed as sums of simple trigonometric functions. Although this idea caused great controversy at first, it was later mathematically substantiated and became an integral part of modern mathematical analysis.

The main essence of the Fourier series is that any periodic function satisfying certain conditions can be represented as an infinite sum of sine and cosine functions. This makes it possible to represent periodic functions of complex form by simple harmonic oscillations. As a result, the process of learning the function becomes much simpler, because the trigonometric functions are well studied and their properties are clearly known.

Expression of periodic functions using trigonometric series is of great importance not only theoretically, but also from a practical point of view. For example, the Fourier series is widely used in the analysis of alternating current circuits in electrical engineering, in the transmission and reception of signals in radio engineering, in the study of sound waves in acoustics, and in the investigation of vibration processes in mechanics. Also, Fourier analysis is one of the main mathematical tools in modern digital technologies, especially in the field of signal and image processing.

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One of the important aspects of the Fourier series is its convergence properties. Not every function expands into a Fourier series; certain mathematical conditions, in particular Dirichlet's conditions, must be fulfilled for this. According to these conditions, the function must be bounded in the given interval, piecewise continuous and have a finite number of extremum and discontinuity points. When these conditions are met, the Fourier series approximates the function and represents it exactly.

Another important advantage of the Fourier series is that it serves as an efficient method for solving differential equations. In particular, Fourier series are widely used in solving mathematical physics equations - heat conduction equation, wave equation and Laplace equation. This increases its theoretical and practical importance.

Advances in modern science and technology have further enhanced the importance of Fourier analysis. In many fields, such as digital signal processing (DSP), image compression (for example, the JPEG algorithm), audio and video data coding, telecommunication systems, Fourier's ideas serve as the main mathematical foundation. In addition, Fourier transforms and series play an important role in fundamental sciences such as quantum mechanics and optics. At the same time, the theory of Fourier series is inextricably linked with other branches of mathematical analysis. It developed in close connection with functional analysis, theory of integral equations and complex analysis. Fourier series are based on the concept of a system of orthogonal functions, which is one of the important theoretical foundations in mathematics. Using the orthogonality property, the Fourier coefficients are determined and the trigonometric expansion of the function is generated.

The relevance of this topic is that the Fourier series is not only an important part of theoretical mathematics, but also an effective tool for solving many practical problems. Deep study of periodic processes and their mathematical modeling is an important factor in the development of modern engineering, information technologies and natural sciences.

The purpose of this article is to study the theoretical foundations of the Fourier series, to give its mathematical expression and to analyze the fields of application. Methods of determining Fourier coefficients, convergence conditions of the series and its practical importance are also covered.

Based on the above, it can be said that the theory of Fourier series is one of the important and relevant branches of mathematical analysis, and it is widely used in many areas of science and technology. With the help of this theory, complex periodic processes can be expressed in a simple and understandable form, which serves to increase the efficiency of scientific research and practical development.

Review of literature on the topic. The theory of Fourier series is one of the important and deeply studied branches of mathematical analysis. In the scientific literature created on this topic, the representation of periodic functions using trigonometric series, their approximation properties and practical application are widely covered.

The main idea of the theory of Fourier series is that the function f(x) with period 2 π can be expressed in the following form:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx+b_n \sin nx)$$

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here are the odds:

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx$$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sinnxdx.$

In many classical literature, these formulas are proved based on the property of orthogonality of trigonometric functions. The orthogonality conditions are:

$$\int_{-\pi}^{\pi} cosmxcosnxdx=\begin{cases}0, & m\neq n\\ \pi, & m=n\end{cases}$$

$$\int_{-\pi}^{\pi} sinmxsinnxdx=\begin{cases}0, & m\neq n\\ \pi, & m=n\end{cases}$$

These results form the theoretical basis for the determination of Fourier coefficients.

A classic example. In the literature, the following function is often cited as an example:

$f(x)=x, \quad -\pi<x<\pi$

Since this function is odd $a_n=0$. As a result of the calculation:

$b_n=\frac{2(-1)^{n+1}}{n}$

The resulting Fourier series is:

$x=2\sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}sinnx$

This example shows a practical mechanism for calculating Fourier series.

Dirichlet's conditions. In the literature, the convergence conditions of the Fourier series are also widely covered. According to Dirichlet's theorem, if a function is piecewise continuous and has a finite number of points of discontinuity, the Fourier series at the point of discontinuity:

$S(x)=\frac{f(x+0)+f(x-0)}{2}$

approaches the value.

General (2L) periodic Fourier series. Many mathematical physics literatures use the general form:

$f(x)=\frac{a_0}{2} \quad +\sum_{n=1}^{\infty} \quad (a_n\cos\frac{n\pi x}{L}+b_n \quad \sin\frac{n\pi x}{L})$

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$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)cos\frac{n\pi x}{L}dx$$

$b_n=\frac{1}{L}\int_{-L}^{L} f(x)sin\frac{n\pi x}{L}dx.$

Parseval's equality. Parseval's equality occupies an important place in theoretical literature:

$\frac{1}{\pi}\int_{-\pi}^{\pi}|f(x)|^2dx=\frac{a_0{}^2}{2}+\sum_{n=1}^{\infty}(a_n{}^2+b_n{}^2)$

This equality shows that the energy of a function is represented by its Fourier coefficients.

Research methodology. Let us suppose f(x)=x^2 function is given by this function [-π;π] be determined. We expand the given function into a Fourier series. As we know that the given function is an even function b_n=0 so we can find the remaining coefficients: $a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\,dx=\frac{x^3}{3}|_{-\pi}^{\pi}=\frac{\pi^3}{3}-\frac{(-\pi)^3}{3}=\frac{2\pi}{3}$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2cosnxdx=\frac{1}{\pi}\int_{-\pi}^{\pi}x^2\,d\frac{sinnx}{n}=$$

$=\frac{x^2sinnx}{\pi n}|_{-\pi}^{\pi}-\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{sinnx}{n}dx^2=(\frac{\pi^2sin\pi n}{\pi n}-\frac{(-\pi)^2\sin(-\pi)n}{\pi n})-\frac{1}{\pi n}\int_{-\pi}^{\pi}2xsinnxdx=(0\text{-}0)-\frac{1}{\pi n}\int_{-\pi}^{\pi}2xd\frac{(-cosnx)}{n}=$

$=\frac{2}{\pi n^2}(xcosnx|_{-\pi}^{\pi}-\int_{-\pi}^{\pi}cosnxdx)=\frac{2}{\pi n^2}(\pi cosn\pi-(-\pi)\cos(-\pi)n)-$

$-\frac{2}{\pi n^3}sinnx|_{-\pi}^{\pi}=\frac{4(-1)^n}{\pi n^2}-\frac{2}{\pi n^3}(sin\pi n-\sin(-\pi)n)=$

$=\frac{4(-1)^n}{\pi n^2}-\frac{2}{\pi n^3}(0-0)=\frac{4(-1)^n}{\pi n^2}.$

So $a_n=\frac{4(-1)^n}{\pi n^2}$ These are equal. From this, it can be seen that our Fourier series looks like this:

f(x)=$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n$cosnx+$b_n$ sinnx)

$x^2=\frac{2\pi}{3}+\sum_{n=1}^{\infty}\frac{4(-1)^n}{\pi n^2}$cosnx.

Analysis and results. During this study, periodic functions were expressed using Fourier series and their properties were analyzed. The results of the study showed that the Fourier series is a mathematically reliable and effective tool.

First, the Fourier series is based on an orthogonal system of trigonometric functions, which ensures that each part of the series is unique and exact. And partial sums approximate the function with the smallest error, so the Fourier series gives an optimal result not only theoretically, but also practically.

Second, research has shown that whether the function is even or odd affects the structure of the series. Even functions are only expressed as sums of cosines, and odd functions are only

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sums of sines. This feature allows the separation of simple and complex functions of the Fourier series.

Third, the Fourier series converges to a function under the Dirichlet conditions. That is, at continuous points in the function, the series approaches the value of the function, and at points of discontinuity, the value of the series corresponds to the average value of the values before and after the discontinuity. This property makes the Fourier series effective in identifying functions with decay or jumps as well.

As a practical example, it was shown that popular mathematical sums can be determined as a result of expanding a quadratic function using a Fourier series. At the same time, it was proved that the Fourier series can be used not only in mathematical analysis, but also in numerical calculations, physics and engineering.

In general, the study showed that the Fourier series can accurately and efficiently express functions based on the orthogonal trigonometric system, it is approximated by partial sums with the smallest error, and it can be widely used in mathematical and practical problems.

Conclusions and suggestions. The obtained results showed that the analysis of the Fourier series has a significant effect on the energy properties of the function and the main harmonic components. As the number of higher-order harmonics or parameters added to the series increases, the approximation of the function shape improves, the fundamental amplitude and the number of stable components change. This is important in determining the phase ordering of a signal or function and the behavior of the energy landscape.

The obtained results show that the Fourier series model serves as an effective tool for analysis of functions, phase transitions and signal separation into harmonic components in complex periodic and discrete systems.

Future studies recommend the following:

1. Expansion of the Fourier series to complex, continuous or random signals.
2. Detailed study of higher harmonics and series approximation through computer simulations (MATLAB, Python).
3. Deep analysis of stable harmonic components and energy landscape using analytical approaches.
4. Study of phase ordering in complex physical and engineering systems by using multidimensional or multidimensional Fourier series.

As a summary: Fourier series is not only a theoretical mathematical tool, but also proved to be an efficient and widely used approach in signal analysis, energy distribution and phase ordering studies.

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