ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23
American Academic publishers, volume 6, issue 05,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai

BINARY RELATIONSHIPS

Department of Mathematics,
University of Economics and Pedagogy
Assistant: **Jamalov Madamin Habibulla ugli**
jamolovmadamin7@gmail.com

https://doi.org/10.5281/zenodo.19978449

Higher-Order Linear Differential Equations with Constant Coefficients

Abstract: The article briefly discusses higher-order linear differential equations with constant coefficients and their solution methods.

Keywords: Higher-order linear differential equation, constant coefficients, characteristic equation.

INTRODUCTION

Binary relations are one of the important concepts of mathematics, which are widely used to express the connections between objects. Binary relations are especially important in the fields of set theory, algebra, and discrete mathematics. With their help, connections between various objects are organized, compared and analyzed.

In the development of modern science and technology, the importance of binary relations is increasing. For example, in computer science, this concept is used to determine the relationship between objects in data structures, algorithms, and programming processes. Therefore, the in-depth study of binary relations is important not only from a theoretical point of view, but also from a practical point of view.

This article provides detailed information about the nature of binary relations, their types and properties, and their application in various fields.

ADABIYOTLAR TAHLILI VA METOD

Given two non-empty sets A and B. We get an element a belonging to set A and an element b belonging to set B. We create an ordered pair (a,b) with the first element a and the second element b.

It consists of all pairs of the form (a, b). $\{(a,b)|a\in A, b\in B\}$ the set is called the Cartesian (exact) product of sets A and B and is denoted as AxB.

Definition 1:$AxB \in R^2$ then the Cartesian product consists of the set of all points in the plane.

Example 1.$A\in [0,1]$ and $B \in [1,2]$ Let's take sets of segment points. This is the Cartesian product of sets

$$AxB=\{(x\ y,)\mid 0 \le x \le 1,\ 1\le y \le 2\}$$

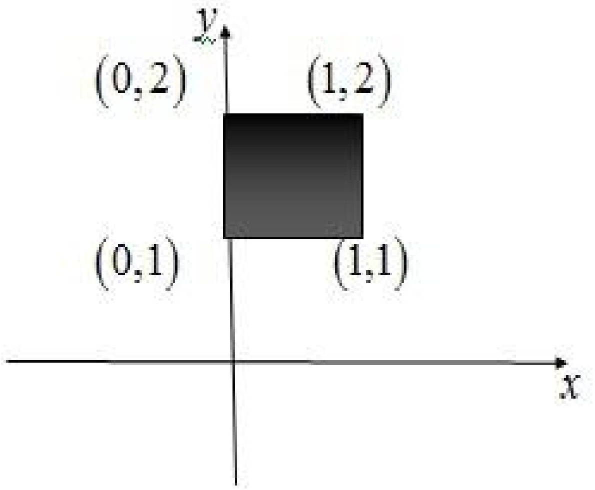
the set will be the set of square points depicted in Figure 5:

ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23
American Academic publishers, volume 6, issue 05,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai



Drawing 1.

It should be noted that two pairs (a,b) and (c,d) are considered equal only if a = c, and b = d.

Cartesian product of several such sets $A_1xA_2xA_3x...xA_n$ we can look like If $A_1=A_2=...=A_n$ if there is, then briefly their Cartesian product $A^n = AxAx...xA$ can be written in the form and is called the Cartesian product of the nth order. A^n the length of the elements is equal to n (x_1, x_2,...,x_n), $x_i \in A$ consists of a string element.

definition 2.A x B An arbitrary subset R of a set is called a binary relation between sets A and B.

In particular, if A = B, R ⊂ A x A binary relation A is called a binary relation defined in A. Binary relations are usually denoted by letters such as R, P, Q.

If R ⊂ AxA binary relationship is defined (x,y) ∈ R then element x is said to be in relation R with element y and xRy defined as

definition 3. A If the following conditions are met for a binary relation R defined in a set, an equivalence relation is said to be defined in a set A: x ∈ A for xRx attitude is appropriate (reflexivity); xRy if the appropriateness of the yRx relationship follows from the relationship (symmetrical);

xRy and yRz from relationships xRz if it turns out that the relationship is appropriate (transitivity).

The equivalence relation R between the elements x and y of the set A is short $x\overset{R}{\sim}y$ written in the form

For example, equality relations on the set of real numbers are equivalence relations.

Theorem 1. An arbitrary equivalence relation R defined on a non-empty set A divides the set A into disjoint classes, and conversely, if the set A is partitioned into disjoint classes, then an equivalence relation corresponding to the given partitions on the set A can be defined.

Proof. Suppose an equivalence relation R is defined on a set A. Optional $a \in A$ for the item $R[a] \in \{x \in A | (a,x) \in R\}$ we define a set. R because it is reflexive $a \in R[a]$ that is, the

ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23

American Academic publishers, volume 6, issue 05,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai

defined set is not empty. We show that these sets form disjoint classes in A. let's say R[a] and R[b] let the sets have a common element. In that case z ∊ R[∊]∩R ∊ [b], i.e z∊ R[a] and z ∊ R[b]. And from that (a,z) ∊ R and (b,z) ∊ R we form that Optional x ∊ R[a] Let's take an element, then (a,x) ∊R. If (a,z) ∊ R using the fact that R is symmetric and transitive (z,x) ∊ R we will make it. Then (b,z) ∊R taking into account (b,x) ∊ R we get This isx ∊ R[b] means that So, R[a] ⊆ R[b].

Just like that R[a] ⊆ R[b] forming that R[a] =R[b] we will have equality. This means that R[a] are non-intersecting classes.

Conversely, if the set A is expressed as a union of non-intersecting classes, we define the relation R as follows. If elements a and b belong to the same class, we say that they are connected by a binary relation R. Obviously, this relation R is an equivalence relation.

Agar biror A to‘plam R ekvivalentlik munosabati yordamida o‘zaro kesishmaydigan qism to‘plamlarga bo‘lingan bo‘lsa, hosil bo‘lgan qism to‘plamlarni ekvivalent sinflar deb ataymiz. A ning bu ekvivalentlik sinflar to‘plami A/R kabi belgilanadi va u faktor-to‘plam deb ataladi.

If a set A is divided into disjoint subsets using the equivalence relation R, the resulting subsets are called equivalence classes. This set of equivalence classes of A is denoted as A/R and is called a factor-set. For example, ℤ all even numbers in the set $\mathbb{Z}_0=\{2n \mid n \in \mathbb{Z}\}$and odd numbers $\mathbb{Z}_1=\{2n+1 \mid n \in \mathbb{Z}\}$ if we divide it into two classes, the equivalence relation corresponding to this division

R ={(x y,)| x - y couple } ⊂ ℤ **x** ℤ it will look like.

DISCUSSION

The concept of binary relations occupies an important place in various branches of mathematics, in particular, set theory, discrete mathematics, and logic. In the process of studying this topic, it was observed that the main properties of binary relations - reflexivity, symmetry, antisymmetry and transitivity - are of particular importance. Through these properties, it will be possible to categorize relationships and determine the scope of their practical application.

During the research, it was found that binary relations are widely used not only theoretically, but also in practical issues. For example, binary relations are one of the main tools in sorting problems, graph theory, database design, and algorithm development. The ability to divide sets into classes using equivalence relations and determine the hierarchy between elements through order relations is especially important.

At the same time, in some cases, it can be difficult to determine the properties of binary relations. Checking all their properties requires additional calculations, especially when working with large collections or when relationships are presented in tabular or graphical form. This makes it necessary to use effective algorithms and automated tools.

Another important point is that systems modeled by binary relationships represent real-life processes in a simplified manner. Therefore, the accuracy of the model depends on how well the selected relationship fits. The wrong attitude can lead to wrong conclusions.

In general, the theory of binary relations serves as an important tool in developing mathematical thinking, building logical analysis skills, and understanding complex systems.

ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23
American Academic publishers, volume 6, issue 05,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai

In the future, a more in-depth study of this topic, especially researching its applications in the fields of computer science and artificial intelligence, will be one of the relevant directions.

Example 1. A set of real numbers $\mathbb{R}$ at x=y the equality relation is a binary relation.

Example 2. A ={2, 5, 4, 6} let it beR ={(x,y)| x < y} a set is a binary relation. Obviously, in this case R ={(2,4), (2,5), (2,6), (4,5), (4,6), (5,6)}.

RESULTS

Theorem 2. If R is a binary relation defined on a set A that is reflexive, antisymmetric, and transitive, then R is a partial order relation.

CONCLUSION

In this article, the concept of binary relations, their main properties and types were considered in detail. As a result of the study, it was found that properties such as reflexivity, symmetry, antisymmetry and transitivity are important criteria in the analysis of binary relations. With these properties, relations can be divided into important classes such as equivalence and order relations.

It was also noted that binary relations are not only theoretical, but also of great practical importance. They are widely used in algorithms, databases, graph theory and many other fields. This further strengthens the position of this topic in modern science and technology.

In summary, binary relationships are an important tool in developing mathematical thinking, modeling complex systems, and building logical reasoning. In the future, conducting more in-depth research in this direction, especially expanding the scope of their practical application, will be one of the urgent tasks.

LIST OF REFERENCES

1. Rosen K. H. Discrete Mathematics and Its Applications. — New York: McGraw-Hill, 2012.

2. Grimaldi R. P. Discrete and Combinatorial Mathematics: An Applied Introduction. — Boston: Pearson, 2004.

3. Coleman B., Busby R., Ross S. Discrete Mathematical Structures. — New Jersey: Prentice Hall, 2009.

4. Halmos P. R. Naive Set Theory. — New York: Springer, 1974.

5. Lipschutz S., Lipson M. Schaum's Outline of Discrete Mathematics. — New York: McGraw-Hill, 2009.

6. Meiliyev H.J., Eshankulov J.C., Jamolov.M.Kh. ``Traektorii kvalrptichnie stochasticheskie operatory na lekartnogo proezvelenie S1*S1. Scientific reports of Bukhara State University. 79-86 c.

7. M. Kh. Jamolov. "Configurations on the Grid", Military Aviation Institute "Actual problems and solutions of science education", conference materials collection, 2025 p.379-383.

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-515-x, Impact Factor: 9.23

American Academic publishers, volume 6, issue 05,2026

Journal: https://www.academicpublishers.org/journals/index.php/ijai

8. M.Kh. Jamolov, "Application of Gibbs Measurements in Lattice Systems", "World Scientific and Methodological Journal of Research", 61(6), p. 162-165.

9. Teshabayev A.T., Khudoyberganov G'A. Ordinary differential equations. - Tashkent: Teacher, 2000.

10. Rasulov H.R. Theory of ordinary differential equations. - Tashkent: Science, 2005.

11. Pontryagin L.S. Ordinary differential equations. - Moscow: Nauka, 1970.