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ANALYTICAL VIEW OF EXACTLY ONE CLASS SURECTIVE QUADRATIC STOCHASTIC OPERATOR.

Department of Mathematics,
University of Economics and Pedagogy
Docent: **Meyliyev Xabibulla Jamolovich**
master:**Sayyod Kuvvatovich Eshmurodov**
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Annotation: In this proverb, each equilateral triangle is self-congruent π_i - It is shown that the surjective quadratic stochastic operators of the exact form belonging to the first class correspond to

Keywords: Self-consistency, surjective, quadratic, stochastic, operators, simplex, side, element, set, homeomorphism, point, Volteto, type, reflection.

In the process of passing from one generation to another, it is desirable to study the class of quadratic stochastic operators and the group of self-adjustments corresponding to it, that is, one self-adjustment corresponds to each class of surjective quadratic operators. $S^{n-1} = \left\{ x = \left(x_1, x_2, ..., x_n \right),\ x_1 + x_2 + ... + x_n = 1.\ x_i \quad 0, i = \overline{1, n} \right\}$

the set is called an n-1-dimensional simplex.

Description.

$$(\mathrm{V}x)_k = x'_k = \sum_{i,j=1}^{n} P_{ij,k} x_i x_j, \mathrm{k} = 1, 2, ..., n \qquad (1)$$

here $\quad \mathrm{P}_{ij,k} \quad 0., \mathrm{P}_{ij,k} = \mathrm{P}_{ji,k}., \sum_{k=1}^{n} \mathrm{P}_{ij,k} = 1$

to S^{n-1} the self-transcending reflection of a simplex is called a quadratic stochastic operator.

We consider the case where n=4 in this problem. In that case $S^3 = \ (x_1, x_2, x_3, x_4) : \mathrm{x}_i \quad 0, , i = 1, 2, 3, 4., \sum_{i=1}^{4} \mathrm{x}_i = 1$ An arbitrary stochastic quadratic operator in the simplex is given by:

$$(\mathrm{V}x)_k = \sum_{i,j=1}^{4} P_{ij,k} x_i x_j, \mathrm{k} = 1, 2, 3, 4$$

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here $\mathrm{P}_{ij,k} \quad 0., \mathrm{P}_{ij,k} = \mathrm{P}_{ji,k} ., \ {}_{k=1}^{4} \ \mathrm{P}_{ij,k} = 1$

With this operator, the following operator is single-valued.

$$P_{11,1}, P_{22,1}, P_{33,1}, P_{44,1} \ P_{12,1}, P_{13,1}, P_{14,1}, P_{23,1}, P_{24,1}, P_{34,1}$$

$$P_{11,2}, P_{22,2}, P_{33,2}, P_{44,2} \ P_{12,2}, P_{13,2}, P_{14,2}, P_{23,2}, P_{24,2}, P_{34,2}$$

$$P_{11,1}, P_{22,3}, P_{33,3}, P_{44,3} \ P_{12,3}, P_{13,3}, P_{14,3}, P_{23,3}, P_{24,3}, P_{34,3}$$

$$P_{11,4}, P_{22,4}, P_{33,4}, P_{44,4} \ P_{12,4}, P_{13,4}, P_{14,4}, P_{23,4}, P_{24,4}, P_{34,4}$$

Here $0 \quad P_{ij,k} \quad 1; i, j, k = 1,2,3,4$.

We define a class of 24 surjective quadratic stochastic operators, and the operators of this class can replace all other surjective quadratic operators. To describe this class of operators, we use the group of self-adjoining polygons [1]. So, S^3 consists of a regular tetrahedron (Fig. 1).

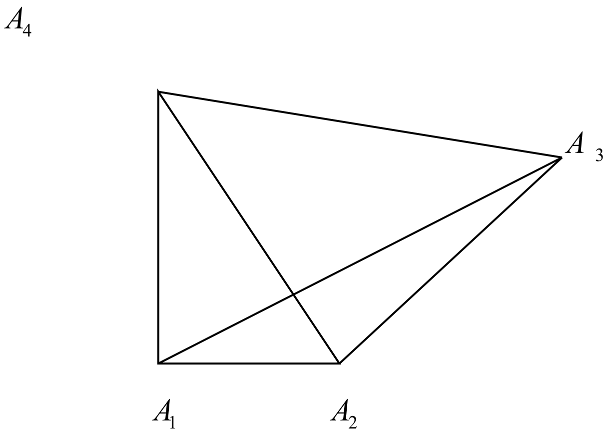

Figure 1.

Self-adaptation refers to a metric-preserving substitution.

R^3 The tetrahedron self-alignment group consists of 12 elements.

We have this issue R^4 We have this issue R^4 in it consists of complete permutations of all vertices of the tetrahedron, i.e

$$G = \{\pi_i\}_{i=1}^{24}.$$

An arbitrary surjective quadratic operator is some self match π_i we prove that corresponds to

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Theorem 2: S^3 self-adjusting surjective quadratic operator defined in $\pi_i,\ i=\overline{1,24}$ corresponds to

Proof. The proof of this theorem follows from the following 3 lemmas;

Lemma 1. Let V be a surjective quadratic operator.

In that case S^3 - no internal point of the simplex will pass to the point at the end of the simplex under the influence of reflection V.

Proof: Assume the reverse

$A=(x_1,x_2,x_3,x_4)\quad IntS^3$ va $V(A)=A_i\ i=1,2,3,4.$

let it be here A_i we consider the simplex for the point at the end (the rest of the cases are proved in the same way). $V(A)=A_i$ the equality follows:

$$1=a_1x_1^2+b_1x_2^2+c_1x_3^2+d_1x_4^2+2\alpha_1x_1x_2+2\beta_1x_1x_3+2\gamma_1x_1x_4+2\zeta_1x_2x_3+2\eta_1x_2x_4+2\delta_1x_3x_4.$$

$$0=a_2x_2^2+b_2x_2^2+c_2x_2^2+d_2x_4^2+2\alpha_2x_1x_2+2\beta_2x_1x_3+2\gamma_2x_1x_4+2\zeta_2x_2x_3+2\eta_2x_2x_4+2\delta_2x_3x_4$$

$$0=a_3x_1^2+b_3x_2^2+c_3x_2^2+d_3x_4^2+2\alpha_3x_1x_2+2\beta_3x_1x_3+2\gamma_3x_1x_4+2\zeta_3x_2x_3+2\eta_3x_2x_4+2\delta_3x_3x_4.$$

$$0=a_4x_1^2+b_4x_2^2+c_4x_2^2+d_4x_4^2+2\alpha_4x_1x_2+2\beta_4x_1x_3+2\gamma_4x_1x_4+2\zeta_4x_2x_3+2\eta_4x_2x_4+2\delta_4x_3x_4$$

$x_1\ x_2\ x_3\ x_4>0$ from

$$a_1=b_1=c_1=d_1=\alpha_1=\gamma_1=\zeta_1=\eta_1=\delta_1=1$$
$$a_i=b_i=c_i=d_i=\alpha_i=\gamma_i=\zeta_i=\eta_i=\delta_i=0$$

It turns out that Dan. That is, voluntary $A\quad S^3$ from $V(A)=A_1$ from $V(S^3)=A_1$ originates. This contradicts the specificity of the quadratic stochastic operator.

Lemma 2: Let V be a surjective quadratic operator. In that case

S^3 each internal point of the simplex does not pass to the boundary points of the simplex under the influence of reflection.

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Proof: *We assume the reverse* $A = (x_1, x_2, x_3 x_4) \quad Int\, S^3$ that is $x_1 \;\; x_2 \;\; x_3 \;\; x_4 > 0$ let it be., i.e. for clarity $V(A) \quad [A_3, A_4]$ let it be

In that case
$$x_1^{'} = 0 = a_1x_1^2 + b_1x_2^2 + c_1x_3^2 + d_1x_4^2 + 2\alpha_1x_1x_2 + 2\beta_1x_1x_3 + 2\gamma_1x_1x_4 + 2\zeta_1x_2x_3 + 2\eta_1x_2x_4 + 2\delta_1x_3x_4.$$

$$x_2^{'} = 0 = a_2x_2^2 + b_2x_2^2 + c_2x_2^2 + d_4^2 + 2\alpha_2x_1x_2 + 2\beta_2x_1x_3 + 2\gamma_2x_1x_4 + 2\zeta x_2x_3 + 2\eta_2x_2x_4 + 2\delta_2x_3x_4.$$

From

$$a_1 = b_1 = c_1 = d_1 = \alpha_1 = \gamma_1 = \zeta_1 = \eta_1 = \delta_1 = 0$$
$$a_2 = b_2 = c_2 = d_2 = \alpha_2 = \gamma_2 = \zeta_2 = \eta_2 = \delta_2 = 0$$

will be. It's optional $A \quad S^3$ for the point $V(A) \quad [A_3, A_4]$,

that is $V(S^3) \quad [A_3, A_4]$ which contradicts the surreactivity of the quadratic stochastic operator V.

Lemma 3: Let V be a surjective quadratic operator. In that case, no boundary point (other than the end of the simplex) will move to the point at the end of the simplex under the influence of reflection.

Proof: $A = (x_1, x_2, x_3 x_4) \quad \partial S^3 \quad A \quad A_i, \quad i = 1,2,3,4$ and for clarity $A \quad [A_1, A_2],\ V(A) \quad A_i\ i = 1,2,3,4$ we need to prove that

Let's assume the opposite, i.e $V(A) = A_i$ let it be, then
$$1 = a_1x_1^2 + b_1x_2^2 + c_1x_3^2 + d_1x_4^2 + 2\alpha_1x_1x_2 + 2\beta_1x_1x_3 + 2\gamma_1x_1x_4 + 2\zeta_1x_2x_3 + 2\eta_1x_2x_4 + 2\delta_1x_3x_4.$$

$$0 = a_2x_2^2 + b_2x_2^2 + c_2x_2^2 + d_4^2 + 2\alpha_2x_1x_2 + 2\beta_2x_1x_3 + 2\gamma_2x_1x_4 + 2\zeta x_2x_3 + 2\eta_2x_2x_4 + 2\delta_2x_3x_4.$$
$$0 = a_4x_1^2 + b_4x_2^2 + c_4x_3^2 + d_4x_4^2 + 2\alpha_4x_1x_2 + 2\beta_4x_1x_3 + 2\gamma_4x_1x_4 + 2\zeta_4x_2x_4 + 2\eta_4x_2x_4 + 2\delta_4x_3x_4$$

from these equations $a_i = b_i = c_i = d_i = \alpha_i = \gamma_i = \zeta_i = \eta_i = \delta_i = 0\ i = 2,3,4$

Therefore, it is optional $A \quad [A_1, A_2],\ V(A) = A_1$ that is $V([A_1, A_2]) = A_1$

which contradicts the surreactivity of the quadratic operator. Thus, the surjective quadratic operator transfers the internal point of the simplex to the internal point, and the boundary point to the boundary points.

Proof of the theorem **lemma** 1., **lemma** 2. and **lemma** 2. comes from .

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The surjective quadratic operator transfers the points at the ends of the simplex to the ends, and the points on the edges to the edges, that is, the surjective quadratic operator is a self-adjustment π_i - as it corresponds to .

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