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DETERMINATION OF MECHANICAL PARAMETERS OF INDENTOR-LIKE ROTATING BODIES PENETRATING INTO ELASTIC-PLASTIC MEDIA BEHIND THE FRONT OF STRONG SHOCK WAVES

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Annotation. Based on his experimental research, the author determined the Lamé parameters $\lambda(\varepsilon,\varepsilon_i)$ and $G(\varepsilon,\varepsilon_i)$ and modeled a homogeneous soil mass as an elastic-plastic continuous medium. By studying the combined effect of normal and tangential stresses, he investigated the propagation of strong shock waves in cylindrically symmetric media, determined the mechanical parameters during the penetration of indenter-like bodies moving with initial velocity and rotation into such media, and obtained new scientific solutions.

Key words. Elasto-plastic medium**,** penetration of rotating indenter-shaped bodies into the surrounding medium**,** explosion**,** mechanical stresses**,** modeling**,** shock wave front**.**

Introduction.

Research combining theory and practice, aimed at the efficient utilization of energy generated by directed and purposeful powerful explosions, has remained relevant since the last century. This research encompasses processes such as excavation and soil transportation in engineering practice, the construction of underground structures, the creation of cavities and gas storage facilities, mining operations, ensuring seismic safety and stability of dams and land reclamation and hydraulic structures, as well as the safe landing of aircraft.

An introduction to the science of the theories of academician Kh.A.Rakhmatulin, including "Rakhmatulin waves" and "layer motion," as well as the model of an ideal "plastic gas," is of great importance for engineering practice. This has become possible due to the research conducted by Kh.A.Rakhmatulin, S.S.Grigoryan, A.Ya.Sagomonyan, S.S.Davydov, G.M.Lyakhov, S.S.Vylov, N.A Tsytoivich, Jacques de Marre, G.I.Pokrovsky, A.Tate, N.V.Maevsky, N.A.Zabudsky, Yu.V.Khaydin, V.A.Veldanov, V.V.Balandin, V.A.Koronatov, V.N.Aptukov, I.V.Khromov, K.S.Sultanov, B.M.Mardonov and A.N.Nabiev. These studies encompassed the investigation of the laws governing the propagation of shock waves, as well as the theoretical modeling and experimental research of the process of penetration of rotating indenter-type bodies into deformable media. The results of these works have found wide application in engineering practice [1,2,3,4,5,6,7,10,12,14].

The mechanical-mathematical essence of the problem

An indenter in the form of a rigid body with an initial velocity $\upsilon_0\neq0$ rotates around its axis of symmetry with an angular velocity $n\neq0$ [rpm] and penetrates into soil or rock masses modeled as an elastic-plastic medium (Fig. 1).

It is required to analyze this penetration process from a mechanical-mathematical point of view and to find solutions to problems such as determining the penetration law – penetration depth, velocity, and pressure.

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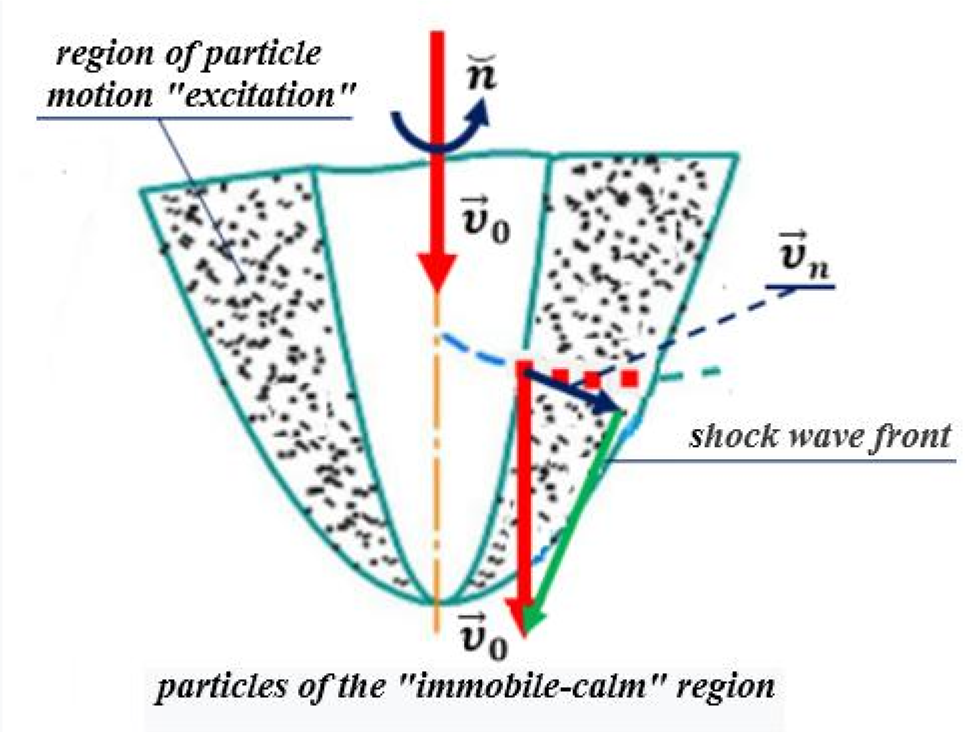

Fig. 1. Penetration of a rotating body into a medium with initial velocity

Typically, in a general case, the solution to such problems is reduced to the investigation and solution of one-dimensional shock waves in elastic-plastic media with cylindrical symmetry from a mechanical-mathematical point of view [1,2,9,10,11].

In the article, in accordance with the deformation theory of "Finite elastic-plastic media", using the results of experiments carried out directly by the author, the motion of indenter-like bodies rotating at a speed of n [rev/sec] (or $N\neq 0$) with the help of a special mechanical drive is studied, for a medium with a coefficient of adhesion of particles k=0, a coefficient of sliding friction $\mu_0\neq 0$, an angle of internal friction of the medium $\vartheta\neq 0$ ($\mu = sin\vartheta$, $\tau_0 = 2kcos\vartheta$) penetrating into media where there is no internal friction.

Methods.

Before solving the problem, taking into account the physical and mechanical nature of the medium and its homogeneity, we rely on the following limitations [2, 8, 13, 14]:

- the law of penetration of rotating bodies is expressed through $H(t)$, with the onset of the penetration process at any moment in time t, the immersing body acquires a velocity $\dot{H}(t)$, and its upper part is also completely located at a depth of $H(t)$;
- the inner boundary of the motion field is a circle of radius $H(t)$, which completely describes the line of intersection of the motion of the immersing body with the plane of the medium;
- Only when shock waves occur does the density of the medium change, and this is determined by the intensity of the wave's motion;
- At the front of shock waves created by impacts, only a stressed state occurs in the media;
- Shock waves propagate in the media at a velocity exceeding the plastic wave velocity.

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A practically important aspect of these limitations is that they provide grounds for concluding that the density of the medium behind the shock waves is a continuous function of the Lagrangian coordinate *r*, independent of time.

The equation for one-dimensional dynamic ground motion in Lagrangian variables during strong explosion processes with cylindrical symmetry is obtained in a more generalized form, taking into account the Prandtl plasticity condition and the combined action of normal and shear stresses [8, 9, 10, 11, 12]:

$$\rho_0\, r\frac{\partial^2 u}{\partial t^2}=(r+u)\frac{\partial\sigma_r}{\partial r}+(\sigma_r-\sigma_\varphi)\frac{\partial}{\partial r}(r+u)+\frac{\partial\tau_{r\varphi}}{\partial\varphi}\cdot\frac{\partial}{\partial r}(r+u) \qquad (2)$$

In general, to solve the problem, taking into account the continuity equation of the form

$$\frac{1}{2}\cdot\frac{\partial}{\partial r}(r+u)^2=\frac{\rho_0}{\rho(r)}\cdot r \qquad (3)$$

it is necessary to integrate the partial differential equation.

However, the above system of equations is not closed, since it contains four unknown functions: the stresses σ_r and σ_φ, the displacement u, and the density of the medium ρ. Therefore, based on the constraint "On the existence of an elastic potential $U(\varepsilon,\varepsilon_i)$", it is recommended to use the Lamé coefficients [1, 2, 9, 10, 12] in practice:

$$G(\varepsilon,\varepsilon_i)=\frac{1}{3\varepsilon_i}\cdot\sigma_i(\varepsilon,\varepsilon_i); \qquad \lambda(\varepsilon,\varepsilon_i)=\frac{1}{3\varepsilon}\cdot\sigma(\varepsilon)-\frac{2}{9}\cdot\frac{1}{\varepsilon_i}\cdot\sigma_i(\varepsilon,\varepsilon_i) \qquad (4)$$

In accordance with the law of conservation of mass and the theorem on the change in momentum, taking into account the boundary conditions and the continuity equation drawn up for a front with a strong shock wave, as well as the fact that the motion begins from the coordinate $r_0=0$, the possibility of transforming the equation of motion with parameters characterizing the internal friction of the medium $\mu=\vartheta=0$ is substantiated as follows [2,8,14]:

$$-\sigma_r=\rho_0\left(\dot R^2+R\ddot R\right)\int_0^{r^*}\frac{rdr}{(2\psi(r)+R^2)}-\rho_0\left(R\dot R\right)^2\cdot\int_0^{r^*}\frac{rdr}{(2\psi(r)+R^2)^2}+\frac{\rho_0}{1-b\left(r^*\right)}\cdot\frac{\left(R\dot R\right)^2}{r^{*\,2}}+$$
$$2\tau_0\cos 2\varphi\left\{\ln\frac{r^*}{R}\right\}+p_\alpha+\left[\frac{\sigma\left(\varepsilon^*\right)}{\varepsilon^*}+\frac{4}{9}\cdot\frac{\sigma_i\left(\varepsilon^*,\varepsilon_i^*\right)}{\varepsilon_i^*}\right]\varepsilon_i^* \qquad (5)$$

Typically, for the case of cylindrical symmetry, firstly, the following relationships hold between the density of the medium, the radial displacement, the relative deformation and the intensity of deformation at the front of strong shock waves:

$$\varepsilon^*=1-\frac{\rho_0}{\rho^*}=u_r^*\ ;\ \varepsilon^*=1-b\left(\rho^*\right);\ \varepsilon_i^*=\frac{2}{3}\cdot\varepsilon^*=\frac{2}{3}\cdot\left(1-\frac{\rho_0}{\rho^*}\right) \qquad (a)$$

Secondly, the Lagrange coordinate function is expressed as follows:

$$\psi(r)=\int_{r_0}^{r}\frac{\rho_0\, r}{\rho(r)}\,dr \quad \text{and} \quad \psi\left(r^*\right)=\int_{r_0}^{r^*}\frac{\rho_0\, r}{\rho(r)}\,dr \qquad (b)$$

According to the law of parity of tangential stresses and the notation $\mu_\beta=(1-\mu_0\tan\beta)^{-1}$, the normal σ_β and tangential τ_β stresses on the inclined surfaces of a particle isolated from a medium subjected to a stress-strain state under the influence of a immersing body are equal to [14]:

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$$\sigma_\beta = \mu_\beta\left(\sigma_r + \frac{C_0 N}{n} tan\beta\right); \qquad \tau_\beta = \mu_0\mu_\beta\left(\sigma_r + \frac{C_0 N}{n}\cdot tan\beta\right) \tag{6}$$

Here $N = N_{kvt}$ is the power expended on the movement of a penetrating body of circular cross-section with an external diameter D at a rotational speed n [rpm]; $C_0 \approx 48657{,}33 \cdot D^{-3}$ is a coefficient depending on the diameter of the penetrating body; β is the angle between the z -axis of symmetry of the particle and the tangent to the generatrix of the penetrating body; μ_0 is the coefficient of sliding friction (in practice it is chosen equal to $\mu_0 \approx 0{,}2 \div 0{,}25$).

The particle $ds = 2\pi \cdot f(x) \cdot \sqrt{1 + f'^2(x)} \cdot dx$, mentally isolated from the medium, is acted upon by the following forces in the normal and tangential directions, respectively:

$$dF = -\sigma_\beta \cdot ds \cdot \sin\beta; \qquad dQ = -\tau_\beta \cdot ds \cdot \cos\beta \tag{d}$$

Then we get the following relationship:

$$dF + dQ = -2\pi\mu_\beta(\sin\beta + \mu_0\cos\beta)\left(\sigma_r - \frac{C_0 N}{n} tan\beta\right) f(x)\sqrt{1 + f'^2(x)}dx \tag{7}$$

The equation of the generatrix (“oral cavity”) of a body penetrating into a medium, $R = f(x)$ at a penetration time $t \geq t_1$ is equal to:

$$R_1 = f(x) = f[H(t) - H_1(t_1)] \tag{i}$$

In processes of strong explosions or when penetrating bodies act on media without internal friction at high speeds, the normal stress in the radial direction is governed by the relation $\sigma_r \cong -p + p_\alpha$ (where p is the pressure in the section $H_1(t_1)$, mentally separated from the medium, and $p_\alpha \cong 1\ kg/cm^2$ is the atmospheric pressure).

After appropriate simplifications, (7) can be written as follows:

$$dF + dQ = 2\pi\frac{\mu_0 + f'(x)}{1 - \mu_0 f'(x)} \cdot \left[(p - p_\alpha) - \frac{C_0 N}{n} f'(x)\right] f(x)dx \tag{8}$$

For the case where the indenter bodies, turning into a cylinder at a height h, penetrate the medium with an opening angle of 2β, the following relationships can be written:

$$R_1 = \tan\beta\, x = \tan\beta\,[H(t) - H_1(t_1)], \quad \dot{R}_1 = \tan\beta\,\dot{H}, \quad \ddot{R}_1 = \tan\beta\,\ddot{H} \tag{9}$$

According to the above reasoning and the notation $a = (1 - b_1)^{-1}$, formulas (8) and (5) are expressed as follows:

$$F + Q = 2\pi\tan^2\beta\frac{1 + \mu_0 \cdot c\tan\beta}{1 - \mu_0\tan\beta}\int_0^H \left[(p - p_\alpha) - \frac{C_0 N}{n}\tan\beta\right] xdx \tag{10}$$

$$p - p_\alpha = \frac{\rho_0}{2b_1}\ln a \cdot f(x) \cdot f'(x) \cdot \ddot{H} + + \frac{\rho_0}{2b_1}\left\{\ln a\left[f(x) \cdot f''(x) + f'^2(x)\right] + b_1 \cdot f'^2(x)\right\} \cdot \dot{H}^2 + \tau_0\cos 2\varphi\ln a + \frac{2}{3}\left\{\sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i\left[(a^{-}\right.\right.$$

Let us determine the pressure from the last formula using the relations (9):

$$p - p_\alpha = \frac{\rho_0}{2b_1}\ln a \cdot \tan^2\beta \cdot x \cdot \ddot{H} + \frac{\rho_0}{2b_1}(\ln a + b_1)\tan^2\beta \cdot \dot{H}^2 +$$
$$2\tau_0\cos 2\varphi\frac{1}{2}\ln a + + \frac{2}{3}\left\{\sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i\left[(a^{-1}),\ \frac{2}{3}(a^{-1})\right]\right\} \tag{11, a}$$

From mechanics it is known that the equation of dynamic motion of a body of mass m is written as follows:

$$m\ddot{H} = -(F + Q) \tag{12}$$

Having introduced the following notation $X = \mu_\beta(1 + \mu_0 \cdot c\tan\beta)$, taking into account expression (10) and the relation $C_0 N n^{-1} = const$, after appropriate simplifications we reformulate

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equation (12) for the immersion of indenter bodies rotating with the initial velocity into the studied medium:

$$m\ddot{H}+\frac{\pi\rho_0\tan^4\beta\ln a}{3b_1}XH^3\ddot{H}+\frac{\pi\rho_0\tan^4\beta}{2b_1}(\ln a+b_1)XH^2\dot{H}^2=-\pi\tan^2\beta X\Big\{\tau_0\cos 2\varphi\ln a+\frac{2}{3}\Big\{\sigma(a^{-1})+\frac{2}{3}\cdot\sigma_i\Big[(a^{-1}),\ \frac{2}{3}(a$$

Now, taking into account the relations $\ddot{H}=y'/2$, $\dot{H}^2=y$, we introduce the following notations:

$$\omega=\frac{\pi\rho_0\tan^4\beta\ln a}{3mb_1};\quad \alpha=\frac{\pi\rho_0\tan^4\beta}{b_1}(\ln a+b_1);\quad \frac{\alpha}{\omega}=3\left(1+\frac{b_1}{\ln a}\right)$$

$$c=\frac{2\pi\tan^2\beta}{m}\Big\{\tau_0\cos 2\varphi\ln a+\frac{2}{3}\Big\{\sigma(a^{-1})+\frac{2}{3}\cdot\sigma_i\Big[(a^{-1}),\ \frac{2}{3}(a^{-1})\Big]\Big\}-\frac{C_0N}{n}\tan\beta\Big\}$$

Accordingly, the following first-order linear differential equation is obtained:

$$y'+\frac{X\alpha H^2}{1+X\omega H^3}\cdot y=-\frac{XcH^2}{1+X\omega H^3} \tag{13}$$

The solution to this equation is as follows:

$$y=-\left[\frac{c}{\alpha}\cdot(1+X\omega H^3)^{\frac{\alpha}{3\omega}}+k_0\right]\cdot\frac{1}{(1+X\omega H^3)^{\frac{\alpha}{3\omega}}} \tag{14}$$

From the initial condition of the problem it is easy to determine k_0:

$$H=0:\quad y_0=\dot{H}_0^2=\upsilon_0^2\quad or\ \ \upsilon_0^2=k_0-\frac{c}{\alpha};\quad k_0=\upsilon_0^2+\frac{c}{\alpha} \tag{15}$$

As a result, the velocity $\dot{H}$ and acceleration $\ddot{H}$ of the penetrating body are easily calculated using the following formulas:

$$y=\dot{H}^2=\frac{\upsilon_0^2+\frac{c}{\alpha}}{(1+X\omega H^3)^{\frac{\alpha}{3\omega}}}-\frac{c}{\alpha}\quad and\ \ \ddot{H}=-\frac{\alpha X}{2}\cdot\frac{\left(\upsilon_0^2+\frac{c}{\alpha}\right)\cdot H^2}{(1+X\omega H^3)^{\frac{\alpha}{3\omega}+1}} \tag{16}$$

During the study, the velocities and stresses at the impact front were determined for the following specific cases of a body penetrating with an initial velocity into a rotating medium or into a rotationally stationary medium, respectively. Generally, for cases of no sliding friction ($\mu_0=0$) and no sliding friction μ_0 and internal friction τ_0 (Fig. 2).

The following data were used for the numerical analysis of the penetration of rotating indenter bodies into a soil mass:

$$N_{dv}=75.5\ kvt,\quad n=300\frac{aln}{min},\quad D=0.25\ m,\quad \upsilon_0=600\frac{m}{sek},\quad h=0.13\ m,$$

$$\vartheta=\frac{\pi}{9},\quad \rho_0=152.9\cdot 10^{-8}\frac{g\cdot sek^2}{sm^4},\quad mg=10\ kg,\quad \mu=sin\vartheta=0.342,$$

$$\tau_0=0.94\frac{kg}{sm^2},\quad \beta=\frac{\pi}{6},\quad k_1=2.683\cdot 10^3\frac{kg}{sm^2},\quad k_2=-4.991\cdot 10^3\frac{kg}{sm^2},$$

$$k_3=-5.744\cdot 10^3\frac{kg}{sm^2},\quad k_4=2.498\cdot 10^3\frac{kg}{sm^2},\ \mu_0=0,2.$$

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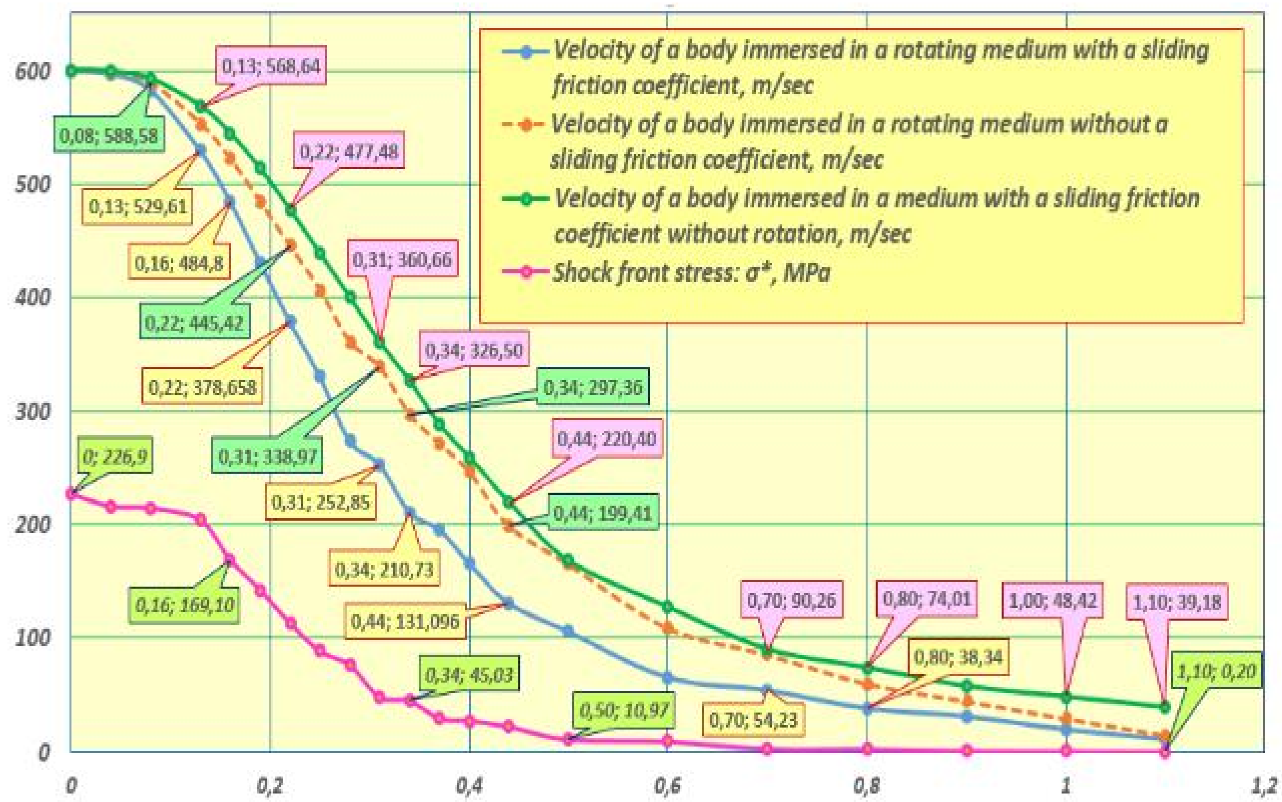

Fig. 2. Dependence of penetration velocity and shock front stress on penetration depth (m)

MAIN CONCLUSIONS

The following scientific solutions and conclusions were reached during the study:

- The study of the laws governing the penetration of solid bodies rotating around a symmetrical axis with an initial velocity, based on the theory of "Finite Elastic-Plastic Deformation," was translated into an examination of the propagation of shock waves in cylindrical symmetrical media.
- Based on the author's experimental research, the Lamé coefficients $\lambda(\varepsilon,\varepsilon_i), G(\varepsilon,\varepsilon_i)$, were refined in form and content and used in modeling soil masses as elastic-plastic continuous media.
- Taking into account the rotational motion of penetrating bodies and the shear stress in the medium, and based on the hypothesis "On the existence of an elastic potential $U(\varepsilon,\varepsilon_i)$", an equation of dynamic motion was derived for the first time, describing the penetration process from a mechanical-mathematical perspective.
- When comparing the results of this study with scientific conclusions and solutions obtained from existing research [1, 2, 4, 5, 6, 7, 14], it was confirmed that the patterns of change in the mechanical parameters of a penetrating indenter body coincide in form and content, as shown in the graphs.

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