



VARIOUS METHODS OF PROVING MATHEMATICAL INEQUALITIES

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Annotation. In this article, various methods of proving inequalities are presented, and examples are given of proving inequalities by definition, artificial methods of proving inequalities, and proving inequalities by inverse hypothesis.

Key words: inequality, support inequality, medium harmonic, medium geometric, medium arithmetic, of numbers medium square.

Establishing interrelationship, connection and consistency between types of education is carried out on the basis of ensuring coherence between subjects, chapters, topics, educational materials. Therefore, it is necessary to ensure continuity between courses covering the basics of mathematics, to select educational materials in accordance with the age characteristics of students of different levels of educational institutions, to place them in a certain logical sequence, interdisciplinary coherence and consistency, it creates the problem of achieving the leading position of the principle of unity in the educational process and pedagogically justifying this situation. In mathematics inequalities to prove circle example and issues often meeting stands. So it is them to prove methods to know very important.

I. Prove inequalities using the definition.

To the definition according to $a > b$ to be for $a - b$ be a positive number need. That's why for given a, b, \dots, k values in the collection $f(a, b, \dots, k) > g(a, b, \dots, k)$ inequality to prove for $f(a, b, \dots, k) - g(a, b, \dots, k)$ difference we will build and he a, b, \dots, k of given in values positive that trust harvest we do. Same also this from the method $f < g, f \geq g, f \leq g$ inequalities it is also used in proof.

Example 1. $a > 0, b > 0$ if, $\frac{a+b}{2} \geq \sqrt{ab}$ (Koshi inequality). prove it

Proof. $\frac{a+b}{2} - \sqrt{ab}$ difference we will build and his hint let's find out.

$$\frac{a+b}{2} - \sqrt{ab} = \frac{a - 2\sqrt{ab} + b}{2} = \frac{(\sqrt{a} - \sqrt{b})^2}{2}$$

$\frac{(\sqrt{a} - \sqrt{b})^2}{2}$ expression optional negative didn't happen a and b in values negative it won't be. So,

$\frac{a+b}{2} \geq \sqrt{ab}$. Equality sign $a = b$ appropriate at.

Example 2. If $ab > 0$ if $\frac{a}{b} + \frac{b}{a} \geq 2$ to be prove it

Proof . $\frac{a}{b} + \frac{b}{a} - 2 = \frac{a^2 + b^2 - 2ab}{ab} = \frac{(a-b)^2}{ab}$. $ab > 0$ that it was for $\frac{(a-b)^2}{ab} \geq 0$. Equality sign

$a = b$ appropriate at . So , $\frac{a}{b} + \frac{b}{a} - 2$ difference negative it's not . It has been proven .

Example 3. This $a^2 + 4b^2 + 3c^2 + 14 > 2a + 12b + 6c$ inequality prove it

Proof . $(a^2 + 4b^2 + 3c^2 + 14) - (2a + 12b + 6c)$ difference we build His terms let's group .

$$(a^2 - 2a + 1) + (4b^2 - 12b + 9) + (3c^2 - 6c + 3) - 1 = (a-1)^2 + (2b-3)^2 + 3(c-1)^2 + 1.$$

The last one expression optional a, b, c appropriate at . It has been proven .

Example 4. If $a + b + c = 0$ if $a^3 + b^3 + c^3 = 3abc$ to be prove it

Proof . $a^3 + b^3 + c^3 - 3abc$ difference let's look . $a^3 + b^3$ expression get together to the cube we will fill it .

$$a^3 + b^3 + c^3 - 3abc = a^3 + 3a^2b + 3ab^2 + b^3 + c^3 - 3a^2b - 3ab^2 - 3abc = (a+b)^3 - 3ab(a+b+c) + c^3$$

Now $(a+b)^3 + c^3$ the to multipliers we separate .

$$\begin{aligned} (a+b)^3 + c^3 - 3ab(a+b+c) &= ((a+b)+c)((a+b)^2 - (a+b)c + c^2) - 3ab(a+b+c) = \\ &= (a+b+c)(a^2 + 2ab + b^2 - ac - bc + c^2 - 3ab) = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac) = \\ &= \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac) = \frac{1}{2}(a+b+c)((a-b)^2 + (a-c)^2 + (b-c)^2) \end{aligned}$$

Condition according to $a + b + c = 0$. Second bracket inside is also negative it's not . Hence , the expression negative it's not . In inequality equality sign $a + b + c = 0$ or $a = b = c$ appropriate at will be

II. Inequalities to prove artificial methods .

this method essence as follows :

Proving inequality one row form of substitutions after from certain (base) inequalities to one is brought . Base inequalities as for example the following from inequalities is used .

a) $a^2 \geq 0$ b) $\frac{a+b}{2} \geq \sqrt{ab}$, in which $a \geq 0, b \geq 0$ d) $\frac{a}{b} + \frac{b}{a} \geq 2$, in which $ab > 0$

e) $ax^2 + bx + c > 0$, in which $a > 0, b^2 - 4ac < 0$.

Example 5. If $a \geq 0, b \geq 0, c \geq 0, d \geq 0$ if $\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd}$ that prove it

Proof . Base inequality as Koshi inequality we can

$$\frac{\frac{a+b}{2} + \frac{c+d}{2}}{2} \geq \sqrt{\frac{a+b}{2} \frac{c+d}{2}} .$$

Own in turn $\frac{a+b}{2} \geq \sqrt{ab}$ and $\frac{c+d}{2} \geq \sqrt{cd}$ that it was for

$$\sqrt{\frac{a+b}{2} \frac{c+d}{2}} \geq \sqrt{\sqrt{ab} \sqrt{cd}} = \sqrt[4]{abcd} .$$

So , $\frac{\frac{a+b}{2} + \frac{c+d}{2}}{2} \geq \sqrt[4]{abcd}$. But $\frac{\frac{a+b}{2} + \frac{c+d}{2}}{2} = \frac{a+b+c+d}{4}$.

So by doing $\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd}$. Proof analysis by doing the following to the conclusion we will

come In inequality equality sign $a = b, c = d$ and $\frac{a+b}{2} = \frac{c+d}{2}$ when appropriate .

Example 6. This $\frac{n+1}{2}^n > n!$, in which $n \in \mathbb{N}, n > 1$ inequality prove it

Proof. Basis inequalities as the following inequalities we can

$$\frac{n+1}{2} > \sqrt{n}; \frac{(n-1)+2}{2} > \sqrt{(n-1)}; \frac{(n-2)+3}{2} > \sqrt{(n-2)}; \dots;$$

$$\frac{2+(n-1)}{2} > \sqrt{2(n-1)}; \frac{1+n}{2} > \sqrt{1 \cdot n}.$$

These n inequalities multiplied by

$$\frac{n+1}{2}^n \sqrt{(n(n-1)(n-2) \dots 2 \cdot 1)(1 \cdot 2 \cdot 3 \dots (n-1) \cdot n)} = \sqrt{n! \cdot n!} = \sqrt{(n!)^2} = n!$$

the harvest we do So, $\frac{n+1}{2}^n > n!$.

Condition according to $n \in \mathbb{N}$ that it was for Try it support from inequalities the first one only strictly to be can In that case support inequalities increase as a result harvest has been last inequality strictly will be So by doing

$$\frac{n+1}{2}^n > n!.$$

Example 7. If $a > 0, b > 0, c > 0$ if $(a+b+c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$ that prove it

Proof. Base inequalities as the following inequalities we can

$$\frac{a}{b} + \frac{b}{a} \geq 2; \quad \frac{a}{c} + \frac{c}{a} \geq 2; \quad \frac{b}{c} + \frac{c}{b} \geq 2.$$

These inequalities $a=b, a=c$ and $b=c$ when appropriate will be them adding

$$\frac{a}{b} + \frac{b}{a} + \frac{a}{c} + \frac{c}{a} + \frac{b}{c} + \frac{c}{b} \geq 6$$

or

$$\frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c} \geq 6$$

the harvest we do

$$1 + \frac{a+c}{b} + 1 + \frac{b+c}{a} + 1 + \frac{a+b}{c} \geq 9,$$

$$\frac{a+b+c}{b} + \frac{a+b+c}{a} + \frac{a+b+c}{c} \geq 9.$$

Equality sign $a=b=c$ appropriate at.

Example 8. If $n \in \mathbb{N}, n > 1$ if $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} < 1$ to be prove it

$$\text{Proof. } \frac{1}{4} = \frac{1}{2 \cdot 2} < \frac{1}{1 \cdot 2}; \quad \frac{1}{9} = \frac{1}{3 \cdot 3} < \frac{1}{2 \cdot 3}; \quad \frac{1}{16} = \frac{1}{4 \cdot 4} < \frac{1}{3 \cdot 4}; \dots; \quad \frac{1}{n^2} = \frac{1}{n \cdot n} < \frac{1}{(n-1) \cdot n}.$$

These $(n-1)$ inequalities adding

$$\begin{aligned} \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} &< \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{(n-1) \cdot n} = \frac{2-1}{1 \cdot 2} + \frac{3-2}{2 \cdot 3} + \frac{4-3}{3 \cdot 4} + \dots + \frac{n-(n-1)}{(n-1) \cdot n} = \\ &= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n-1} - \frac{1}{n} = 1 - \frac{1}{n} < 1 \end{aligned}$$

the harvest we do So , $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{n^2} < 1$.

III. From the reverse assumption to do method using inequalities to prove

Example 9. If $a > 0, b > 0, c > 0, d > 0$ if $\sqrt{(a+c)(b+d)} < \sqrt{ab} + \sqrt{cd}$ that prove it

Proof . Optional negative didn't happen a, b, c, d numbers for given inequality appropriate that let's prove it need From the reverse assumption we do Negative didn't happen a, b, c, d values for $\sqrt{(a+c)(b+d)} < \sqrt{ab} + \sqrt{cd}$ inequality appropriate let it be This is inequality each two part negative that it was not for his each two part to the square carrying

$$(a+b)(b+d) < ab + cd + 2\sqrt{abcd}$$

inequality harvest we do From this

$$bc + ad < 2\sqrt{abcd} ; \frac{bc + ad}{2} < \sqrt{(bc)(ad)}.$$

But this Koshi to inequality is contradictory . So , ours our hypothesis wrong . That's why for given inequality is correct .

Example 10. If $a > 0, b > 0, c > 0$ if $\frac{a+b+c}{3} > \sqrt{\frac{a^2+b^2+c^2}{3}}$ that prove it

Proof . a, b and c of the following inequalities satisfying negative didn't happen values assuming it exists we do

$$\frac{a+b+c}{3} > \sqrt{\frac{a^2+b^2+c^2}{3}}.$$

Har two part to the square carrying

$$\frac{a+b+c}{3}^2 > \frac{a^2+b^2+c^2}{3}$$

the harvest we do

$$(a+b+c)^2 > 3(a^2+b^2+c^2),$$

$$3(a^2+b^2+c^2) - (a+b+c)^2 < 0,$$

$$3(a^2+b^2+c^2) - (a^2+b^2+c^2+2ab+2ac+2bc) < 0,$$

$$2a^2+2b^2+2c^2-2ab-2ac-2bc < 0,$$

$$(a-b)^2 + (b-c)^2 + (a-c)^2 < 0.$$

The last one inequality appropriate not because squares sum there will be no negative number . So , our hypothesis appropriate it's not . That's why for given inequality appropriate will be

Instruction . n ta a_1, a_2, \dots, a_n negative didn't happen numbers for the following sizes suitable for :

$$H_n = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}, \text{ medium harmonic}$$

$$G_n = \sqrt[n]{a_1 a_2 \dots a_n}, \text{ medium geometric}$$

$$A_n = \frac{a_1 + a_2 + \dots + a_n}{n}, \text{ medium arithmetic .}$$

$$Q_n = \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}}, a_1, a_2, \dots, a_n \text{ of thigs medium square .}$$

These are the sizes between the following relationship appropriate

$$H_n \leq G_n \leq A_n \leq Q_n.$$

This is a relationship private some cases above proved . In examples 1-5 G_2 A_2 and s, G_4 A_4 proved in example 7 and H_3 A_3 example 10 A_3 Q_3 .

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