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FINDING THE ROOTS OF AN EQUATION USING THE CHORD METHOD

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Annotation: The article describes the chord method for approximate solution of algebraic equations. A program for solving the problem in Pascal is provided.

Keywords and expressions: fixed point, zero approximation

Let there be an equation $f(x) = 0$ and the root of the equation x* lies on the segment [a,b], that is $x^* \in [a,b]$.

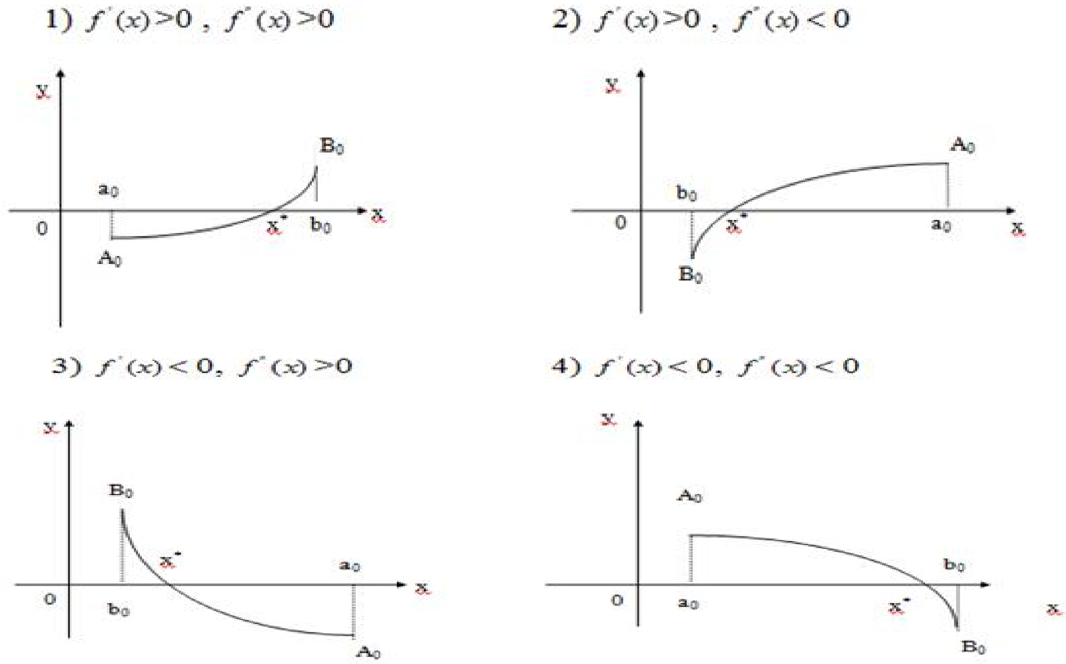
To apply the chord method, the following conditions must be met:

1) The function $f(x)$ is continuous on the interval $[a,b]$ with its first and second derivatives;

2) The function $f(x)$ takes opposite signs at the ends of the segment $[a,b]$, that is $f(a) \cdot f(b) < 0$

3) Derivatives and functions $f(x)$ retain a certain sign on the interval $[a,b]$. This means that the function f(x) is monotone and the root x* is unique.

The geometric meaning of the chord method is that the graph of the function f(x) on the segment [a,b] is replaced by a chord. There can be 4 cases for the function graph:



Let's consider the first case, then the graph of the function has the following form:

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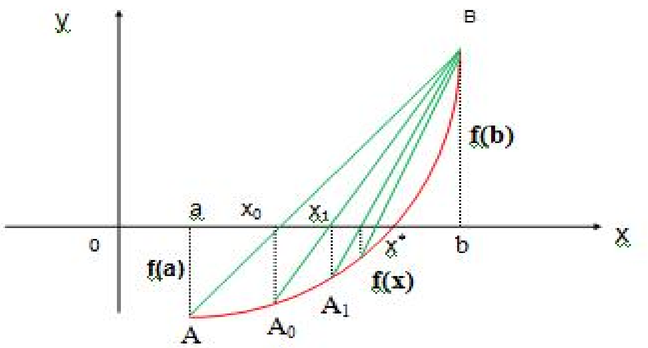

Fig.1. Geometric illustration of the chord method

Let x^* - be the root of the equation and x_0 its approximate value. In order to find the approximate value of x_0, the graph of the function f (x) on the segment [a, b] is replaced by the chord AB. We write the equation of the line passing through the points $A(a, f(a))$ and $B(b, f(b))$:

$$\frac{x-a}{b-a} = \frac{y-f(a)}{f(b)-f(a)} \qquad (1)$$

Since $x_0 \in [a,b]$ lies on the line AB, we can find it from equation (1) by substituting the values $x = x_0$ and $y = 0$::

$$x_0 = a - \frac{f(b)}{f(b)-f(a)}(b-a) \qquad (2)$$

If the condition $f(b)\, f''(b) > 0$ is met, point b is fixed and then formula (2) can be used to find the zero approximation of x_0.

The equation of the straight line passing through points $A(a, f(a))$ and $B(b, f(b))$ can be written in this form:

$$\frac{x-b}{a-b} = \frac{y-f(b)}{f(a)-f(b)} \qquad (1')$$

For $x = x_0$ and we $y = 0$ get:

$$x_0 = b - \frac{f(b)}{f(b)-f(a)}(b-a) \qquad (2')$$

If the condition $f(a)\, f''(a) > 0$ is met, the point a is stationary and then formula (2′) can be used to find the zero approximation of x_0.

To find the approximation x_1, we write the equation of the line A_0B and for x=x1 and y=0 we get:

$$x_1 = x_0 - \frac{f(x_0)}{f(b)-f(x_0)}(b-x_0)$$

From the equation of straight line A_1B at $x=x_2$ and y=0 we obtain:

$$x_2 = x_1 - \frac{f(x_1)}{f(b)-f(x_1)}(b-x_1)$$

Continuing the process, we find the n-th approximation:

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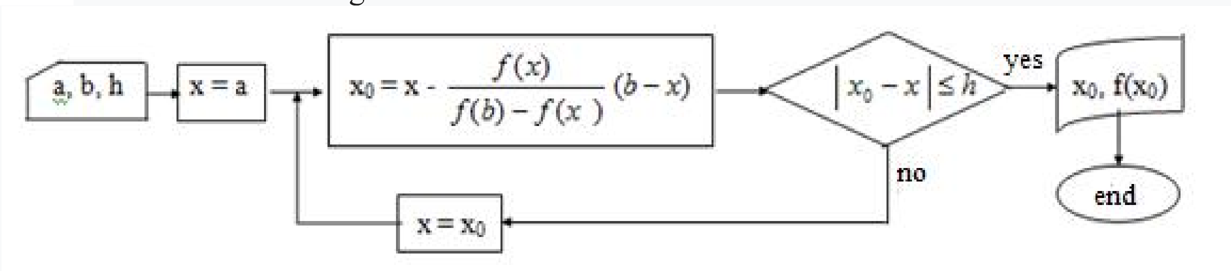
$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f(b) - f(x_{n-1})}(b - x_{n-1}) \qquad (3)$$

Calculations continue until the condition $\left| x_n - x_{n-1} \right| \quad h$ is met.

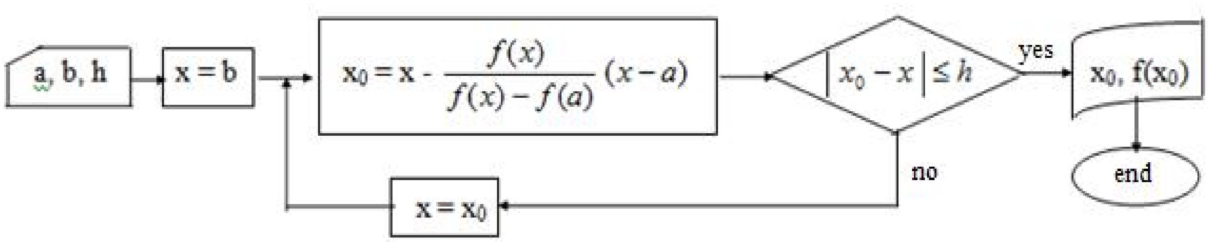
There is a value of n such that $\lim\limits_{n\to} x_n = x^*$ satisfies the condition $\left| x_n - x^* \right| \quad h$ and the calculations stop.

The block diagram of the chord method algorithm is as follows:

1) if point b is fixed, i.e. when the condition $f(b)\ f''(b) > 0$ is met, the block diagram has the following form:



2) if point a is fixed, i.e. when the condition $f(a)\ f''(a) > 0$ is met, the block diagram has the following form:



Example: Find the root of the equation $f(x) = x^4 + 5x^2 + 10x - 3 = 0$ on the segment [0,1] with an accuracy of h=0.001 using the chord method.

Solution: We find the first and second derivatives:

$$f'(x) = 4x^3 + 10x + 10,\ f''(x) = 12x^2 + 10$$

$$f(0)f''(0) < 0 \text{ and } f(1)f''(1) > 0$$

Since $f(1)f''(1) > 0$), point B is fixed and calculations are carried out according to the following formula:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f(b) - f(x_{n-1})}(b - x_{n-1})$$

In this case x_0=a, so we will use the first type of block diagram.

The Pascal program looks like this:

```
program chord method_ B fixed;
var a,b,x,h,x0:real;
label return, answer;
function fx(x:real):real;
begin fx:=x*x*x*x+5*x*x+10*x-3
```

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```
end;
begin
writeln('a='); readln(a);
writeln('b='); readln(b);
writeln('enter precision'); readln(h);
x:=a;
return: x0:=x-fx(x)/(fx(b)-fx(x))*(b-x);
if abs(x0-x)<=h then goto answer
else x:=x0; goto return;
answer:writeln(x0,' ',fx(x0));
end.
```

After executing the program, enter the values 0,1,0.001 sequentially from the keyboard and press the ENTER button each time. We get the following answer:

x= 0.264665, F(x) = 0.797911E-03

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