

USING PREDICATES TO CONSOLIDATE THEORETICAL KNOWLEDGE OF INEQUALITIES FOR FUTURE PRIMARY SCHOOL TEACHER

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Annotation: This article explores the importance of using predicate algebra formulas in teaching students how to prove theorems and solve inequalities. The examples and theorems presented in the article can be used not only for teaching students the applications of predicate algebra, but also for working with students with creative abilities and organizing group lessons in mathematics.

Key words: Inequalities, predicate, range of truth of a predicate, equivalent formulas, theorem, proof of a theorem, methods of proof.

The science of mathematics is studied and developed based on the laws of mathematical logic. However, mathematical logic is not taught as a separate subject in secondary schools. Although the elements of mathematical logic are partially included in mathematics textbooks, their applications are not sufficiently covered. As a result, students face many difficulties in studying the theoretical foundations of mathematics in depth, solving equations and inequalities, and especially in proving theorems. Taking this into account, in this article we will consider the applications of predicate algebra to solving inequalities and systems of inequalities and proving theorems.

When studying the applications of predicate algebra, it is important to know its equivalence formulas. Let us recall the main equivalence formulas of predicate algebra:

$$P(x) \wedge (S(x) \vee Q(x)) \equiv P(x) \wedge S(x) \vee P(x) \wedge Q(x) \quad (1)$$

$$P(x) \vee S(x) \wedge Q(x) \equiv (P(x) \vee S(x)) \wedge (P(x) \vee Q(x)) \quad (2)$$

$$\overline{P(x) \wedge S(x)} \equiv \overline{P(x)} \vee \overline{S(x)} \quad (3)$$

$$\overline{P(x) \vee S(x)} \equiv \overline{P(x)} \wedge \overline{S(x)} \quad (4)$$

$$P(x) \Rightarrow S(x) \equiv \overline{P(x)} \vee S(x) \quad (5)$$

$$P(x) \Rightarrow S(x) \equiv \overline{S(x)} \Rightarrow \overline{P(x)} \quad (6)$$

$$P(x) \Leftrightarrow S(x) \equiv P(x) \wedge S(x) \vee \overline{P(x)} \wedge \overline{S(x)} \quad (7)$$

$$P(x) \Leftrightarrow S(x) \equiv (\overline{P(x)} \vee S(x)) \wedge (\overline{S(x)} \vee P(x)) \quad (8)$$

Since inequalities consist of predicates, the problem of solving an inequality comes down to finding the truth domain of the predicate. Let $P(x)$ and $S(x)$ be predicates defined on some set \mathcal{M} . We denote the truth domains of these predicates by E_p and E_s , respectively, the negation of the predicate $P(x)$ by $\bar{P}(x)$, and the set $\mathcal{M} \setminus E_p$ by \bar{E}_p .

$\bar{P}(x)$, $P(x) \vee S(x)$, $P(x) \wedge S(x)$, $P(x) \Rightarrow S(x)$ and $P(x) \Leftrightarrow S(x)$ We use the following theorems to find the truth domains of the predicates.

1- theorem. $(\forall x \in \mathbb{R})(x^2 \leq x \Rightarrow x \leq \sqrt{x})$.

Proof. We use the converse method: that is, instead of the given theorem, we prove the following theorem $(\forall x \in \mathbb{R})(x \leq \sqrt{x} \Rightarrow x^2 \leq x)$, which is equally strong. Let $(x \geq \sqrt{x} \Rightarrow x^2 \geq x)$ $x \geq \sqrt{x}$. Then $x \geq 1$. We multiply both sides of this inequality by x (Since the value of x is positive, the inequality sign does not change). $x \cdot x \geq x \cdot 1$ means that the inequality $x \geq \sqrt{x}$ implies the inequality $x^2 \geq x$. Thus, the formula $(\forall x \in \mathbb{R})(x \leq \sqrt{x} \Rightarrow x^2 \leq x)$ is a theorem. Therefore, the formula $(\forall x \in \mathbb{R})(x^2 \leq x \Rightarrow x \leq \sqrt{x})$, which is equally strong as this formula, is also a theorem.

2- theorem. $(E_p = E_s) \Rightarrow (\forall x \in \mathcal{M})(P(x) \Leftrightarrow S(x))$

Proof. Let $E_p = E_s$. Then the element $\forall x$ belonging to E_p also belongs to E_s .

At this value of x , $P(x)$ and $S(x)$ are true statements. Therefore, based on the definition of the equivalence operation, the formula $P(x) \Leftrightarrow S(x)$ is also a true statement. Therefore, from the equality $E_p = E_s$ it follows that the formula $\forall x \in \mathcal{M}((P(x) \Leftrightarrow S(x)))$ is a true statement. The theorem is proved.

Let \mathbb{R} be the set of real numbers.

Example 1. Given a predicate $x^2 - 7x + 12 < 0$ in the set \mathbb{R} . Find its truth domain.

Solving. We denote the given predicate by $P(x)$ and its truth domain by E_p . Then, $P(x) \equiv (x^2 - 7x + 12 < 0) \equiv ((x - 3)(x - 4) < 0) \equiv$

$$\equiv (x - 3 < 0) \wedge (x - 4 > 0) \vee (x - 3 > 0) \wedge (x - 4 < 0) \equiv$$

$$\equiv (x < 3) \wedge (x > 4) \vee (x > 3) \wedge (x < 4).$$

$$E_p = (-\infty; 3) \cap (4; \infty) \cup (3; \infty) \cap (-\infty; 4) = \emptyset \cup (3; 4) =$$

$$(3; 4). \text{ Answer: } E_p = (3; 4).$$

Example 2. Given a predicate $P(x) = (x^2 - x - 20 > 0)$ defined on a set \mathbb{R} . Find its truth domain E_p .

$$\text{Solving. } P(x) \equiv (x^2 - x - 20 > 0) \equiv ((x + 4) \cdot (x - 5) > 0) \equiv$$

$$\equiv (x + 4 < 0) \wedge (x - 5 < 0) \vee (x + 4 > 0) \wedge (x - 5 > 0) \equiv$$

$$\equiv (x < -4) \wedge (x < 5) \vee (x > -4) \wedge (x > 5),$$

$$E_p = \{x \in \mathbb{R} | x < -4\} \cap \{x \in \mathbb{R} | x < 5\} \cup$$

$$\cup \{x \in \mathbb{R} | x > -4\} \cap \{x \in \mathbb{R} | x > 5\} = (-\infty; -4) \cap (-\infty; 5) \cup$$

$$\cup (-4; \infty) \cap (5; \infty) = (-\infty; -4) \cup (5; \infty).$$

$$\text{Answer: } E_p = (-\infty; -4) \cup (5; \infty).$$

Example 3. Given a predicate $P(x) = \left(\frac{2x+6}{5x-10} \leq 0\right)$ in a set \mathbb{R} , find its truth domain E_p .

$$\text{Solving. } P(x) = \left(\frac{2x+6}{5x-10} \leq 0\right) \equiv (2x + 6 \leq 0) \wedge (5x - 10 > 0) \vee$$

$$\vee (2x + 6 \geq 0) \wedge (5x - 10 < 0) \equiv (x \leq -3) \wedge (x > 2) \vee (x \geq -3) \wedge (x < 2).$$

$$E_p = \{x \in \mathbb{R} | x \leq -3\} \cap \{x \in \mathbb{R} | x > 2\} \cup$$

$$\cup \{x \in \mathbb{R} | x \geq -3\} \cap \{x \in \mathbb{R} | x < 2\} = (-\infty; -3] \cap (2; \infty) \cup [-3; \infty) \cap$$

$$\cap (-\infty; 2) = \emptyset \cup [-3; 2) = [-3; 2). \text{ Answer: } E_p = [-3; 2).$$

Example 4. Given a predicate $P(x) = (|x-2| < 3)$ in the set \mathbb{R} . Find its truth domain E_p .

$$\text{Solving. } P(x) = (|x - 2| < 3) \equiv (x - 2 < 3) \wedge (x - 2 > -3) \equiv$$

$$\equiv (x < 5) \wedge (x > -1).$$

$$E_p = \{x \in \mathbb{R} | (x < 5) \wedge (x > -1)\} = \{x \in \mathbb{R} | x < 5\} \cap \{x \in \mathbb{R} | x > -1\} =$$

$$= (-\infty; 5) \cap (-1; \infty) = (-1; 5). \text{ Answer: } E_p = (-1; 5).$$

Example 5. Given a predicate $P(x) = (|2x + 6| \geq 4)$ in the set \mathbb{R} . Find its truth domain E_p .

$$\text{Solving. } P(x) = (|2x + 6| \geq 4) \equiv (2x + 6 \geq 4) \vee (2x + 6 \leq -4) \equiv$$

$$\equiv (2x \geq -2) \vee (2x \leq -10) \equiv (x \geq -1) \vee (x \leq -5).$$

$$E_p = \{x \in \mathbb{R} | (x \geq -1) \vee (x \leq -5)\} = \{x \in \mathbb{R} | x \geq -1\} \cup \{x \in \mathbb{R} | x \leq -5\} =$$

$$= [-1; \infty) \cup (-\infty; -5] = (-\infty; -5] \cup [-1; \infty).$$

$$\text{Answer: } E_p = (-\infty; -5] \cup [-1; \infty).$$

Example 6. Given the predicates $P(x) = (x^2 - x \leq 0)$ and $S(x) = (x \leq \sqrt{x})$ defined on a set \mathbb{R} , find $E_p = ?$, $E_s = ?$, $E_{p \wedge s} = ?$, $E_{p \vee s} = ?$, $E_{p \Rightarrow s} = ?$, $E_{s \Rightarrow p} = ?$,

$$E_{p \Leftrightarrow s} = ?$$

Solving.

$$E_p = \{x \in \mathbb{R} | x^2 - x \leq 0\} = \{x \in \mathbb{R} | x(x - 1) \leq 0\} = \{x \in \mathbb{R} | (x \leq 0) \wedge (x - 1) \geq 0\} \vee \{(x - 1) \leq 0\} \wedge \{x \geq 0\}\} = \{x \in \mathbb{R} | x \leq 0\} \cap \{x \in \mathbb{R} | x \geq 1\} \cup \{x \in \mathbb{R} | x \leq 1\} \cap \{x \in \mathbb{R} | x \geq 0\} = (-\infty; 0] \cap [1; \infty) \cup (-\infty; 1] \cap [0; \infty) = \emptyset \cup [0; 1] = [0; 1]; E_p = [0; 1].$$

$$E_s = \{x \in \mathbb{R} | x \leq \sqrt{x}\} = \{x \in \mathbb{R} | (x \geq 0) \wedge (x^2 \leq x)\} = \{x \in \mathbb{R} | x \geq 0\} \cap \{x \in \mathbb{R} | x(x - 1) \leq 0\} = [0; \infty) \cap [0; 1] = [0; 1]. E_s = [0; 1].$$

$$E_{p \wedge s} = E_p \cap E_s = [0; 1] \cap [0; 1] = [0; 1]$$

$$E_{p \vee s} = E_p \cup E_s = [0; 1] \cup [0; 1] = [0; 1]$$

$$E_{p \Rightarrow s} = \bar{E}_p \cup E_s = (-\infty; 0) \cup (1; \infty) \cup [0; 1] = (-\infty; \infty).$$

$$E_{s \Rightarrow p} = (-\infty; \infty). E_{p \Leftrightarrow s} = E_{p \Rightarrow s} \cap E_{s \Rightarrow p} = (-\infty; \infty).$$

The following theorems can be used to teach students how to solve proof problems using equivalence formulas of predicate algebra.

$$3\text{- theorem. } (\forall x \in \mathbb{R})(x \leq \sqrt{x} \Rightarrow x^2 \leq x).$$

$$4\text{- theorem. } (\forall x \in \mathbb{R})(x \leq \sqrt{x} \Leftrightarrow x^2 \leq x).$$

$$5\text{- theorem. } (\forall x \in \mathcal{M})(P(x) \Rightarrow S(x)) \Rightarrow (E_p \subset E_s).$$

$$6\text{- theorem. } E_p \subset E_s \Rightarrow (\forall x \in \mathcal{M})(P(x) \Rightarrow S(x)).$$

$$7\text{- theorem. } (E_p = E_s) \Rightarrow (\forall x \in \mathcal{M})(P(x) \Leftrightarrow S(x)) [6].$$

The examples and problems discussed above can be used to teach students the applications of predicate algebra. When students are taught the laws of mathematical logic, rules of induction, equivalence formulas, and their applications in depth and in detail, their ability to solve mathematical problems in the simplest ways, quickly, and without errors, will develop.

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