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METHODS FOR DEALING WITH EXAMPLES OF SECOND-ORDER CURVES IN ANALYTICAL GEOMETRY

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Annotation: This article shows how to solve non-standard examples of second-order lines. This article can be a great guide for independent learners.

Keywords: Ellipse, parabola, hyperbola, eccentricity, directrix, focal radius, focus, asymptotic.

Ellipse - the locus of points, the sum of the distances from each of which to two given points F_1, F_2 (focus) is a constant value equal to $2a$.

Ellipse elements:
$2a$ - major axis;
$2b$ - semi-minor axis of the ellipse;
A_1, A_2, B_1, B_2 - peaks;
$F_1(c;0), F_2(-c;0)$ - focus;
$2c$ -focal length $c^2 = a^2 - b^2$

$\varepsilon = \frac{c}{a} < 1$ - eccentricity. The eccentricity of an ellipse can be considered as a measure of its "elongation": the greater the eccentricity, the smaller the ratio $\frac{b}{a}$.

$r_1 = a - \varepsilon x, r_2 = a + \varepsilon x,$ - focal radius

$d_1 : x = \frac{a}{\varepsilon}, d_2 : x = -\frac{a}{\varepsilon}$ - directrix.

The canonical equation of the ellipse (coordinate axes coincide with the axes of the ellipse):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Hyperbola - the locus of points, for each of which the modulus of the difference in distances from it to two given points F_1, F_2 (focus) is a constant value equal to $2a$.

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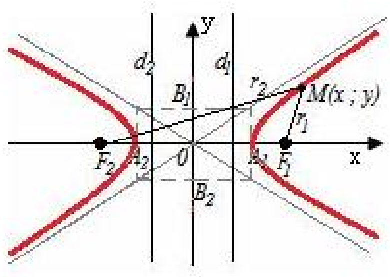
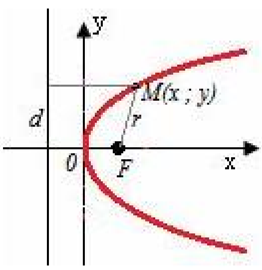


Elements of hyperbole:
$2a$ - real axis;
$2b$ - imaginary axis;
A_1, A_2 - peaks;
$F_1(c;0), F_2(-c;0)$ - focal radius;
$2c$ - focal length (focal length) $c^2 = a^2 + b^2$
$y = \pm \frac{b}{a} x$ - asymptotes;
$\varepsilon = \frac{c}{a} > 1$ - eccentricity $(c > a)$. It can be considered as a numerical characteristic of the magnitude of the angle between the asymptotes.
$r_1 = \pm(\varepsilon x - a),\ r_2 = \pm(\varepsilon x + a),$ - focal radius (the upper sign corresponds to the right branch, the lower sign corresponds to the left branch)
$d_1 : x = \frac{a}{\varepsilon},\ d_2 : x = -\frac{a}{\varepsilon}$ - directrix.
The geometric meaning of the imaginary axis is shown in the figure by a dotted line (the distance between the asymptotes).
The canonical equation of a hyperbola (the coordinate axes coincide with the axes of the hyperbola):

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Parabola - locus of points, each of which is equidistant from a given point (focus) and from a given line (directrix):



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Elements of a parabola:

OF - focal axis;

O - vertex;

$F\ \frac{p}{2};0$ - focus;

$\varepsilon = 1$ - eccentricity;

$r = x + \frac{p}{2},\ (p > 0)$ - focal radius;

$d : x = -\frac{p}{2}$ - directrix;

p - focal parameter.

The canonical equation of a parabola (the Ox -axis coincides with the focal axis, the origin of coordinates coincides with the vertex of the parabola):

$$y^2 = 2px$$

For $p < 0$, the branches of the parabola are directed to the left.

If the focal axis coincides with the Oy axis, then the parabola equation has the form: $x^2 = 2py$

For $p > 0$, the branches of the parabola are directed upwards, for $p < 0$ downwards.

Example Solutions

Task.1. Bring the equation of the 2nd order curve to the canonical form, find all its parameters, construct the curve.

$$9x^2 - 4y^2 - 90x - 8y + 185 = 0.$$

Decision. We bring the equation of the curve to the canonical form, highlighting the perfect squares:

$$9(x^2 - 10x) - 4(y^2 + 2y) + 185 = 0$$

$$9(x^2 - 10x + 25) - 4(y^2 + 2y + 1) = -185 + 225 - 4$$

$$9(x - 5)^2 - 4(y + 1)^2 = 36$$

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$$\frac{(x-5)^2}{4}-\frac{(y+1)^2}{9}=1$$

$$\frac{(x-5)^2}{2^2}-\frac{(y+1)^2}{3^2}=1.$$

We have obtained the canonical equation of the hyperbola $\frac{(x-5)^2}{2^2}-\frac{(y+1)^2}{3^2}=1$ with the center at the point $O(5;-1)$ and the semiaxes $a=2,\ b=3$.

Asymptotes of a hyperbola: $y+1=\pm\frac{3}{2}(x-5)$; $y=\pm\frac{3}{2}(x-5)-1$:

Parameter: $c: c^2=a^2+b^2=4+9=13,\ c=\sqrt{13}$

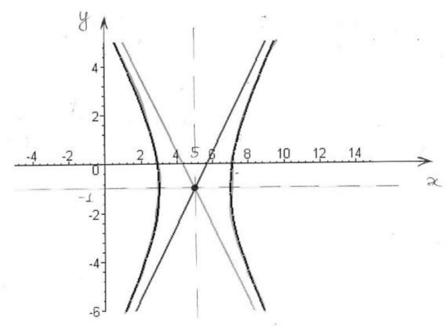
Then the focus of the hyperbola is located at the points:

$F_1(c+5;-1)=F_1(\sqrt{13}+5;-1)$ и $F_2(-c+5;-1)=F_2(-\sqrt{13}+5;-1)$.

Hyperbola eccentricity: $\varepsilon=\frac{c}{a}=\frac{\sqrt{13}}{2}\approx 1{,}8>1$.

Hyperbole directrixes: $x-5=\pm\frac{a}{\varepsilon},\ x-5=\pm\frac{4}{\sqrt{13}}$,

Let's make a drawing. Draw a hyperbola and its asymptotes, mark the center $O(5;-1)$



Task.2. The general equation of the curve of the second order can be reduced to the canonical one. Find the coordinates of the center, the coordinates of the vertices and foci. Write the equations of asymptotes and directrixes. Draw lines on graphs, mark points.

$$9x^2+25y^2-18x-100y-116=0.$$

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Decision. We bring the equation of the curve to the canonical form, highlighting the full squares:

$$9x^2 + 25y^2 - 18x - 100y - 116 = 0.$$

$$(9x^2 - 18x) + (25y^2 - 100y) = 116,$$

$$9(x^2 - 2x) + 25(y^2 - 4y) = 116,$$

$$9(x^2 - 2x + 1) + 25(y^2 - 4y + 4) = 116 + 9 + 100,$$

$$9(x-1)^2 + 25(y-2)^2 = 225,$$

$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$$

$$\frac{(x-1)^2}{5^2} + \frac{(y-2)^2}{3^2} = 1.$$

This is the equation of an ellipse centered at point $O(1;2)$ and semiaxes $a = 5, b = 3$.

Vertices in points

$$A_1 = (1+5;2) = A_1(6;2),$$

$$A_2 = (1-5;2) = A_2(-4;2),$$

$$A_3 = (1;2-3) = A_3(1;-1),$$

$$A_4 = (1;2+3) = A_4(1;5).$$

Axes of symmetry for the curve: $x = 1,\ \ y = 2.$

Ellipse directrixes: $x = \pm\frac{a}{\varepsilon} + 1 = \pm\frac{25}{4} + 1,\quad x_1 = \frac{29}{4},\ x_2 = -\frac{21}{4}.$

Parameter $c: c^2 = a^2 - b^2 = 25 - 9 = 16,\ c = 4$

Eccentricity is $\varepsilon = \frac{c}{a} = \frac{4}{5} = 0.8 < 1.$ Focus points: $F_1(-c+1;2) = F_1(-3;2)$ и $F_2(c+1;2) = F_2(5;2)$.

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Task.3. Given a curve $y^2 + 6x + 6y + 15 = 0$.

1. Prove that this curve is a parabola.
2. Find the coordinates of its vertex.
3. Find the values of its p parameter.
4. Write down the equation of its axis of symmetry.
5. Build this parabola.

Decision. We bring the equation of the curve to the canonical form, highlighting the full squares:

$$y^2 + 6x + 6y + 15 = 0$$

$$(y^2 + 6y) + 6x + 15 = 0,$$

$$(y^2 + 6y + 9) + 6x + 15 - 9 = 0,$$

$$(y+3)^2 = -6x - 6,$$

$$(y+3)^2 = -6(x+1),$$

$$(y+3)^2 = 2\ \ (-3)\ \ (x+1).$$

This is the canonical equation of the parabola $y_1{}^2 = 2\ \ (-3)x_1$ with the parameter $p = -3\ \ (y^2 = 2px)$

The top is at the point $A(-1;-3)$.

Parameter $p = -3$.

Axis of symmetry $y_1 = y + 3 = 0$, that is $y = -3$.