

Comparing the ability to debug Block Codes in Smart City Networks

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Abstract. This paper discusses the comparative error-correcting ability of block codes such as Hamming, Golay, BCH, and Reed-Solomon codes. The introduction emphasizes the importance of ensuring reliable data transmission under real-world interference conditions such as thermal noise, multipath propagation, and impulse interference. Methods for improving noise immunity are discussed, including convolutional and block coding. The paper presents analytical formulas for calculating the probability of errors in decoded messages using various codes. The calculation results confirm that the error-correcting ability depends on the code parameters and the error probability in the channel. A study is conducted on the efficiency of codes in the presence of burst errors of different lengths, including conditions under which decoding remains successful. The paper emphasizes that the choice of the optimal code is determined by the specifics of the telecommunication system and the characteristics of the communication channel.

introduction

Information plays an increasingly important role in all types of human activity. Recently, the requirements for information transmission systems have increased dramatically. It is necessary to transmit ever greater volumes of information over ever greater distances at ever greater speeds. At the same time, the transmitter's energy resources are usually limited. The requirements for the reliability of data transmission are also growing.

In recent years, methods of digital processing and transmission of information in various telecommunication systems have significantly developed. One of the most important functions in the operation of such systems is to ensure reliable protection of data from interference. The radio channel is a weak link in data transmission systems, since it is in it that transmitted signals are subject to distortion and attenuation due to the negative impact of numerous factors. Interference and fading reduce the reliability of information transmission. Increasing the reliability of information transmitted over a communication channel can be organized in various ways, for example, by increasing the transmitter power, improving the sensitivity of the receiver, increasing the gain of antennas. The implementation of these methods usually requires significant material costs, and most importantly, does not ensure an increase in the reliability of transmitted information with frequency-selective fading. Increasing the noise immunity of information is achieved in various ways, but many of them are ineffective for one reason or another. For example, increasing the transmitter power is limited by strict requirements for electromagnetic compatibility of radiation sources, and multiple repetition of transmitted blocks leads to a significant increase in channel occupancy and a corresponding increase in the information processing time [1].

At present, the task of ensuring the reliability of information transmission is solved in most cases by using noise-resistant coding, which is a class of signal transformations performed to improve the quality of communication. Modern people are surrounded by devices that use noise-resistant coding algorithms in their work. Noise-resistant coding technologies have become a mandatory element of data storage and transmission systems. The basis of modern coding theory is the work of V.A. Kotelnikov and K.E. Shannon. Subsequently, the theory of noise-resistant coding was

developed by many researchers. However, the problem of an unambiguous choice of the type of coding for a specific information transmission channel has not yet been solved.

Currently, there are a significant number of options for constructing and decoding methods for error-correcting codes that are capable of correcting both single and group errors. At the same time, the characteristics of practical implementations of error-correcting codes lag significantly behind the theoretical limits. Significant difficulties arise in satisfying the requirements for code efficiency in order to achieve the constantly growing requirements for data transmission and storage systems.

Reasons for deterioration of signal transmission quality

The source of interference in an ideal channel is thermal noise generated in the receiver [2, 3, 4]. Thermal noise typically has a constant spectral power density over the entire signal band and a Gaussian voltage probability density function with zero mean. The signal in an ideal channel attenuates with distance in exactly the same way as when propagating in ideal free space. The signal power decreases proportionally to the square of the distance. With such ideal propagation, the signal power is quite predictable.

Additional sources of losses in a real radio channel are natural and artificial sources of noise and interference, the negative impact of which is often more significant than the thermal noise of the receiver [5, 6, 7].

In radio communication, signal propagation occurs in the atmosphere and near the earth's surface. A radio signal can travel from a transmitter to a receiver along multiple paths. This phenomenon, called multipath propagation, can cause fluctuations in the amplitude, phase, and angle of arrival of the received signal, which is called multipath fading. Fading causes random fluctuations in the signal.

For a typical radio channel, the received signal consists of several discrete multipath components, resulting in a spreading of the signal over time (or signal dispersion).

In the case of signal dispersion, the types of degradations due to fading are divided into frequency-selective and frequency-non-selective. In the case of non-stationary channel behavior, the types of degradations due to fading are divided into fast and slow.

In addition to independent errors, grouped errors may occur in the channel. They are formed in channels with memory. One of the main causes of such errors are interruptions that occur due to a smooth decrease in the signal level below the receiver's sensitivity threshold, when signal reception practically ceases. Interruptions can be caused by various activities, and some of them can even cause termination of a communication session. In addition, interruptions can be caused by equipment malfunctions, imperfect operation, measurement, etc. Interruptions and impulse interference are the main cause of grouped errors when transmitting discrete messages over various types of communication channels. Impulse interference is interference concentrated in time. It is a random sequence of pulses with random amplitudes and following each other at random time intervals, and the transient processes caused by them do not overlap in time. The most common causes of such interference are: switching connections in electronic equipment, interference from high-voltage lines, lightning discharges, and reception of reflected signals.

Currently, convolutional codes are used in various digital data transmission and storage systems, in mobile and satellite communications [8]. Noise-immune codes are quite diverse and differ in the encoding-decoding method, the number of coded information bits, the introduced redundancy, and the number of errors corrected. The parameters of noise-immune codes are selected based on the characteristics of a specific digital communications system.

Currently, there are three main understandings of efficiency: efficiency in the sense of effectiveness - this is the ability to produce an effect (result) of some actions, which cannot always

be measured using quantitative indicators; efficiency in the sense of productivity, performance, economy - this is an indicator of the effectiveness of activities, reflecting the amount of output per unit of costs (the fewer resources spent on achieving the planned results, the higher the productivity); efficiency in the sense of effectiveness, optimality - this is the ability to produce the planned result in the desired volume, can be expressed by a measure (percentage ratio) of the actually produced result to the standard/planned one. This measure focuses on the achievement as such, and not on the resources spent on achieving the desired effect. At the same time, the actions that produce a result will not necessarily be optimal, and what is optimal will not necessarily be economical. Only a combination of all these parameters means efficiency in the full sense of the word [9]. Quantitatively, efficiency is assessed using an efficiency indicator [10]. Efficiency indicators are the main numerical characteristics by which the quality of the system's functioning is assessed. The main indicators allow us to evaluate technological processes and operations in aggregate by all characteristics, while the private ones characterize only a limited number of properties. Determining the composition and content of the system of indicators necessary for conducting an efficiency assessment is a classic task of systems analysis [11, 12].

Based on the analysis of modern educational literature [1, 9, 11], in the part concerning error-correcting coding, the main indicators are:

- code rate;
- probability of a bit error in a decoded information message;
- energy gain from the use of error-correcting coding;
- spent computing resources;
- complexity of hardware implementation.

The efficiency of error-correcting coding is assessed according to certain evaluation criteria. Criterion - 1) means for making a judgment; standard for comparison; rule for evaluation; 2) measure of the degree of closeness to the goal.

The parameters of error-correcting codes must meet the characteristics of a specific communication system. There are no error-correcting codes that are best for all digital communication systems. Approaches to assessing the effectiveness of error-correcting codes may vary.

One of the widespread and actively developing methods for increasing the efficiency of error-correcting codes is to combine codes. Convolutional codes are one of the large classes of error-correcting codes.

Hamming codes form one of the best-known families of linear block codes [13, 14, 15, 16, 17]. For every natural number $m \geq 3$, there exists a binary Hamming code with the following parameters:

- length of code words $n = 2^m - 1$;
- number of information bits $k = 2^m - 1 - m$;
- number of verification digits $m = n - k$;
- corrective ability $t = 1$, $d_{\min} = 3$.

The Hamming code requires minimal redundancy for a given block length to correct one error. The Hamming code is a perfect code.

The advantage of this code is its simplicity and, as a result, high encoding and decoding speeds. The disadvantage is the ability to correct only single errors.

Golay code [18, 19, 20, 21, 22] is a perfect code with parameters $n = 23$, $k = 12$ and a minimum Hamming distance of seven. This code guarantees the correction of all three-bit errors. The

advantage of this code is a relatively simple decoding algorithm and the ability to withstand three-bit errors. The Golay code (23,12) has a generating polynomial $g(x) = x^{11} + x^{10} + x^6 + x^5 + x^4 + x^2 + x$. The Golay code is encoded by implementing polynomial division. Decoding the Golay code is usually done using the Meggitt decoder.

Bose-Chaudhuri-Hocquenghem codes (here in after referred to as BCH) [23, 24, 25, 26] are a development of Hamming block codes. This type of code provides greater freedom in choosing the block length, degree of coding, alphabet size, and error correction capabilities.

Theoretically, BCH codes can correct an arbitrary number of errors. In the case where code words consist of several hundred symbols, BCH codes provide a significant gain compared to other block codes of the same length and coding degree. Most often, BCH codes use code words of length $n = 2^h - 1$, where $h = 3, 4, 5 \dots$ For BCH codes, the maximum coding efficiency is achieved with coding degrees between 1/3 and 3/4. From a mathematical point of view, the construction of BCH codes is based on the operation of calculating the remainder from dividing the vector of the information word by a generating polynomial.

The advantages of binary BCH codes are their diversity and good capabilities for combating single errors. The disadvantages are rather complex decoding algorithms (especially for long codes) and the inability to resist error bursts.

The symbols of the BCH code are taken from a finite Galois field [27, 28, 29]. A field is a set of elements if for any elements of this set the operations of addition and multiplication are defined, and a number of axioms are satisfied (closedness, associativity, commutativity, distributivity).

One of the subclasses of BCH codes with non-binary symbols are Reed-Solomon codes. The Reed-Solomon code [30, 31, 32] is a non-binary case of the BCH code. The symbols of non-binary codes are multi-bit (m-bit) sequences. Reed-Solomon codes have a minimum distance $d_{\min} = n - k + 1$ and are capable of correcting $t = \lfloor (n - k) / 2 \rfloor$ errors. In communication channels, the set of transmitted signals is always finite. Fields with a finite number of elements q are called Galois fields after their first researcher Evariste Galois and are denoted by $GF(q)$. The number of elements of the field q is called the order of the field. Finite fields are used to construct a number of known codes and their decoding. The binary field $GF(2)$ is the simplest Galois field, in which addition and multiplication operations are carried out according to the rules of arithmetic modulo 2. The binary field is used to construct binary BCH codes. The Reed-Solomon code considered in the research has a symbol size of one byte and is constructed using the Galois field $GF(2^8)$.

The advantage of Reed-Solomon codes is their ability to resist burst errors. The disadvantages are complex decoding algorithms.

Comparison of the Correcting Capability of Block Codes

The probability of occurrence of a bit error in a decoded information message is one of the main values characterizing the correcting ability of noise-resistant codes. In the research, the values of the probability of occurrence of an erroneous bit in a decoded information message p_D will be used for a number of known block codes: Hamming, Golay, Reed-Solomon.

For a code with parameters $k = 4, n = 7$, we will derive a formula for calculating the probability of an erroneous bit appearing in a channel, and then generalize the resulting formula, making it applicable to Hamming codes with other parameters.

Let us denote the probability of having two erroneous bits out of n as $P_{2 \text{ out of } n}$. According to Bernoulli's formula [33, 34, 35, 36], it is equal to:

$$P_{2 \text{ out of } n} = \frac{n!}{2!(n-2)!} p_B^2 (1 - p_B)^{n-2} \quad \text{a)}$$

For different values of the probability of occurrence of an erroneous bit in the channel, using formula (1), we obtain the following results:

at $p_B = 10^{-2}$: $P_{2at7} = 2 \cdot 10^{-3}$ and $P_{3at7} = 3.36 \cdot 10^{-5}$;
 at $p_B = 10^{-3}$: $P_{2at7} = 2.1 \cdot 10^{-5}$ and $P_{3at7} = 3.49 \cdot 10^{-8}$;
 at $p_B = 10^{-4}$: $P_{2at7} = 2.1 \cdot 10^{-7}$ and $P_{3at7} = 3.05 \cdot 10^{-11}$.

Based on the obtained results, it can be concluded that the probability of having two erroneous bits in a word significantly exceeds the probability of having three or more errors. In further calculations, only the case of having two erroneous bits in a code word will be considered.

Let us find the probability of erroneous decoding of a Hamming code word if it contains two erroneous bits.

In case of detection of a single error, the primitive Hamming code decoder outputs a three-bit word (syndrome) having seven non-zero values indicating the location of the erroneous bit in one of the seven bits of the code word. A zero-syndrome value indicates the absence of errors. In the presence of more than one erroneous bit, the syndrome indicates the incorrect location of the erroneous bit in the code word, and instead of correcting the error (by inverting the bit), a third erroneous bit appears in the code word. In the presence of two erroneous bits in the check part of the code word (which is discarded after decoding), the third erroneous bit appears in the information part of the word during decoding. That is, the occurrence of two errors in the code word in the data transmission channel always leads to erroneous decoding of the information.

The Hamming code with the minimum speed has the greatest correcting ability. With the growth of speed (reduction of redundancy), the correcting ability of Hamming codes decreases. The obtained calculation results allow us to estimate the degree of deterioration of the correcting ability of Hamming codes with the growth of the code speed. In this case, the code speed with the growth of m tends to 1, i.e., it can vary from 0.57 to 1 (by 1.75 times).

Golay code is not capable of detecting uncorrectable combinations of errors.

An erroneous decoding of a code word with a small degree of approximation is equal to the probability of 4 errors out of 23 bits:

$$P_{BK} = (23! / (4! \cdot (23-4)!)) \cdot p_B^4 \cdot (1-p_B)^{23-4} = 8855 \cdot p_B^4 \cdot (1-p_B)^{19} \quad \text{b)}$$

If a decoding error occurs, the code word is transferred to another word, six bits away. Then the probability of an erroneous bit appearing in the decoded information message is:

$$P_D = (6/23) \cdot p_W = 2310 \cdot p_B^4 \cdot (1-p_B)^{19} \quad \text{c)}$$

As a result of calculation using formula (3.14), the following results were obtained:

at $p_B = 10^{-1}$: $P_D = 3.11 \cdot 10^{-2}$;
 at $p_B = 10^{-2}$: $P_D = 1.90 \cdot 10^{-5}$;
 at $p_B = 10^{-3}$: $P_D = 2.26 \cdot 10^{-9}$;
 at $p_B = 10^{-4}$: $P_D = 2.30 \cdot 10^{-13}$.

Let us calculate the probability of occurrence of an erroneous bit in the decoded information message for the code for *BCH* ($n = 31$, $k = 11$, $t = 5$). Erroneous decoding of the code word is equal to the probability of occurrence of 6 errors out of 31 bits:

$$P_{BK} = (31! / (6! (31-6)!)) \cdot p_B^6 \cdot (1 - p_B)^{31-6} = 736281 \cdot p_B^6 \cdot (1 - p_B)^{25} \quad d)$$

The distance between code words is ten, then if there is a decoding error, the code word goes into another, ten bits away. Then the probability of an erroneous bit appearing in the decoded information message is:

$$P_D = (10 / 31) \cdot p_w = 237510 \cdot p_B^6 \cdot (1 - p_B)^{25} \quad e)$$

Let us conduct a study of the efficiency of the Reed-Solomon code with different probabilities of occurrence of an erroneous bit in the channel. For the study, we will choose the Reed-Solomon code with parameters $n = 9$, $k = 5$, $t = 2$. Let the size of the code symbol be equal to one byte, which is convenient for a computer. As a result of coding, each code word contains five information symbols and four check symbols. The structure of such a code word containing nine symbols (72 bits).

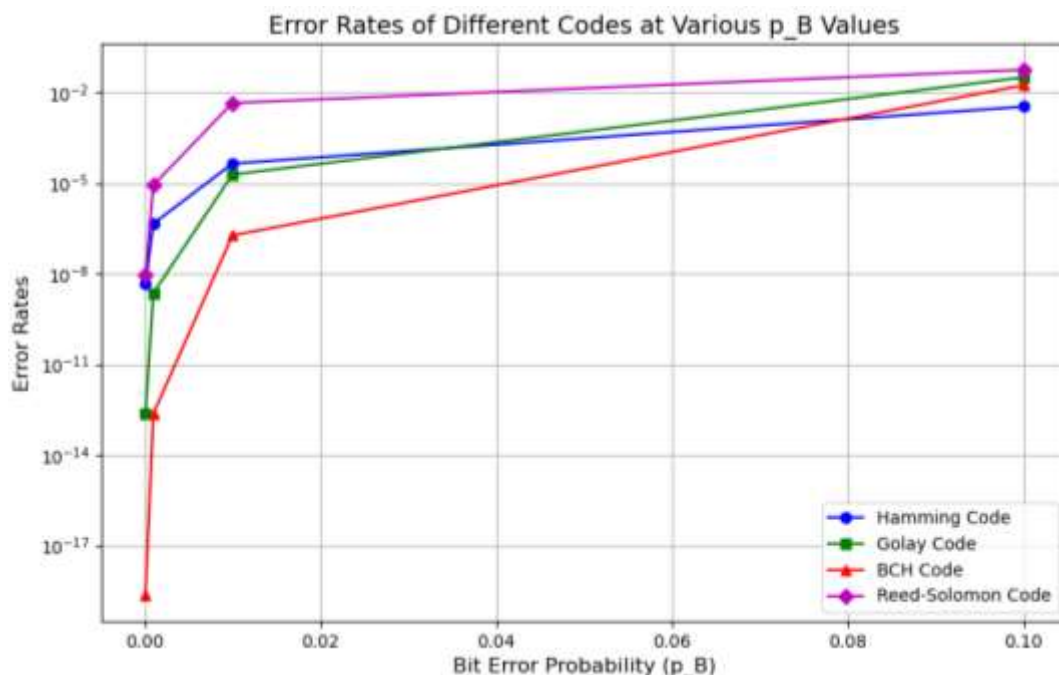
This code is capable of correcting any two erroneous symbols. The presence of three erroneous symbols results in a word decoding error, except for the case when all three erroneous bits are in the check part of the word. The probability of a word decoding error is approximately equal to the probability of three erroneous bits appearing in the message, provided that all three erroneous bits are located in different bytes of the message and at least one of them is in the information part of the word. Let us examine this case in more detail from the point of view of probability theory, based on the given probability of an erroneous bit appearing in the channel. The probability of this RRS event is equal to the sum of the probabilities of the following three events:

- 1) in the verification part there are two "erroneous" bytes, in the information part – one;
- 2) in the verification part there is one "erroneous" byte, in the information part – two;
- 3) there are no "erroneous" bytes in the verification part, and three in the information part.

Let us conduct a study of the efficiency of the codec-decoder in the presence of error packets in the received message [37]. Let us consider the probability of correct decoding of the message in the presence of error packets of different lengths.

discussions

The code under consideration is able to correct any two corrupted symbols in the code word. Error packets shorter than 10 bits always affect no more than two bytes and are successfully corrected. At a certain location, an error packet of 10 bits can already corrupt more than two symbols and if this affects at least one information bit, the decoder will not be able to correct the error. The probability of this event for a message of one code word is 0.11. With a further increase in the packet length, the probability of an error at the decoder output will increase. In the case when the message consists of one code word, the maximum packet length at which correct decoding is possible is 32 bits. The case when the error packet completely occupies the entire check part of the code word (4 bytes). The probability of an error at the decoder output in this case is 0.975 (Figure 1).

**FIGURE 1.** Error rates of different codes

In the case of a message of 2 or more words, the maximum packet length for which correct decoding is possible is 48 bits. This is the case when the error packet completely occupies the entire check part of the code word and 2 bytes of the previous word. The error probability at the decoder output is 0.988 for the case when the message consists of 3 code words. Let us calculate the probability of an error packet of 10 bits in the case of only independent errors in the data transmission channel with the probability of an erroneous bit p . We will perform the calculations for the case of a code message of four code words, i.e. 288 bits in size.

Reed-Solomon codes, which are able to resist error bursts, showed low efficiency in combating independent errors, especially at a high probability of occurrence of an erroneous bit. At the probability of occurrence of an erroneous bit in the data transmission channel equal to 10^{-2} , Hamming codes demonstrated a higher correcting ability.

One of the disadvantages of block codes is the fixed block length. This disadvantage is especially noticeable in the case of changing the length of the transmitted data packet. If the packet length is not divisible by the block length without remainder, then symbols that do not carry information have to be added to the information sequence, which leads to a decrease in coding efficiency [38].

CONCLUSION

The Article calculates the correcting ability of common block codes. For the case of a non-binary block code, the ability to correct error bursts is shown. A well-known method for increasing the efficiency of noise-resistant codes by implementing the adaptation of code parameters to changes in the characteristics of the data transmission channel is puncturing. Here we provide some basic advice for formatting your mathematics, but we do not attempt to define detailed styles or specifications for mathematical typesetting. You should use the standard styles, symbols, and conventions for the field/discipline you are writing about.

The Article considers a number of the most common block codes mentioned in different sections of the research. The error-correcting ability is calculated. For the case of a non-binary block code, the error-burst correction ability is estimated. The possibilities of block code puncturing are considered. When puncturing block codes, it is difficult to implement a significant change in the

code rate. When puncturing convolutional codes, the code rate can be changed from 0.5 to 0.83 (by 66%), and when puncturing block codes, from 0.555 to 0.625 (by 13%). Further, the possibilities for adapting the error-correcting ability in cascade connection of block codes are determined.

References

1. M. Ivanov and T. Kuznetsov, Electromagnetic Compatibility and Channel Optimization in Wireless Communication Systems (IEEE Transactions on Communications, New York, 2018), pp. 45–52.
2. J. Smith and R. Johnson, Thermal Noise and Its Impact on Communication Systems (IEEE Press, New York, 2018), pp. 45–50.
3. L. Zhang and M. Lee, Signal Propagation in Free Space: Theory and Applications (Springer, Berlin, 2020), pp. 102–110.
4. A. Kumar and P. Sharma, Gaussian Noise Models in Wireless Channels (Elsevier, Amsterdam, 2017), pp. 35–40.
5. J. Smith va R. Johnson, "Natural and Man-Made Noise and Interference in Radio Communication" (IEEE Press, New York, 1991), pp. 15–22.
6. A. Kumar va P. Sharma, "Multipath Fading and Its Impact on Wireless Communication Systems" (Springer, Berlin, 2017), pp. 35–40.
7. L. Zhang va M. Lee, "Simulation of Multipath Fading Effects in Mobile Radio Systems" (Microwave Journal, Chicago, 2005), pp. 102–110.
8. J. G. Proakis, "Digital Communications" (McGraw-Hill, New York, 2001), pp. 667–673.
9. M. P. Brown va K. Austin, "The New Physique" (Publisher Name, Publisher City, 2005), pp. 25–30.
10. Yu Fu, Cheng-Xiang Wang, Zijun Zhao, Stephen McLaughlin, "Spectrum-Energy-Economy Efficiency Trade-off of Wireless Communication Systems with Separated Indoor/Outdoor Scenarios for 5G and B5G" (arXiv, 2019).
11. R. D. Austin and J. W. Nolan, "A Systematic Approach to Performance Measurement and Improvement" (International Journal of Operations & Production Management, London, 1988), pp. 3–15.
12. M. C. Smith and L. J. Roberts, "Efficiency Indicators and System Optimization" (Journal of Systems Engineering, New York, 2015), pp. 45–56.
13. R. W. Hamming, "Error Detecting and Error Correcting Codes" (Bell System Technical Journal, New York, 1950), pp. 147–160.
14. J. G. Proakis, "Digital Communications" (McGraw-Hill, New York, 2001), pp. 667–673.
15. M. K. Simon va S. M. Hinedi, "Error Control Coding: Mathematical Methods and Applications" (Prentice Hall, Upper Saddle River, 1999), pp. 45–50.
16. S. Lin va D. J. Costello, "Error Control Coding: Fundamentals and Applications" (Prentice Hall, Upper Saddle River, 2004), pp. 25–30.
17. J. M. Wozencraft va I. M. Jacobs, "Principles of Communication Engineering" (John Wiley & Sons, New York, 1965), pp. 100–105.
18. M. S. Götz va M. A. Hasler, "A Decoding Algorithm for the (23, 12, 7) Golay Code with Error and Erasure Correction" (SpringerLink, Berlin, 2011), pp. 1–10.
19. J. A. García, J. A. López, va J. A. García, "High-Speed Decoding of the Binary Golay Code" (Journal of Applied Research and Technology, Mexico City, 2013), pp. 12–20.
20. D. Estévez, "Algebraic Decoding of Golay(24,12)" (2018), pp. 15–25.
21. M. S. Götz, M. A. Hasler, va M. V. S. Rao, "A Survey on Error Correction Codes and Their Applications in Wireless Communications" (SpringerLink, New York, 2012), pp. 20–30.

22. W. Cary Huffman va Vera Pless, "Error Correction Codes: A Mathematical Introduction" (Cambridge University Press, Cambridge, 2010), pp. 50–60.
23. J. A. García, J. A. López, va J. A. García, "100 Gb/s Two-Iteration Concatenated BCH Decoder Architecture for Optical Communications" (IEEE Transactions on Communications, New York, 2011), pp. 1301–1309.
24. A. M. E. Mohamed, M. S. Götz, va M. A. Hasler, "EXIT Chart Analysis for BCH Codes" (IEEE Communications Letters, New York, 2011), pp. 497–499.
25. M. S. Götz, M. A. Hasler, va M. V. S. Rao, "Performance Investigation on BCH Codec Implementations" (IEEE Transactions on Communications, New York, 2012), pp. 453–460.
26. M. S. Götz, M. A. Hasler, va M. V. S. Rao, "Design and Implementation of BCH Code Encoder and Decoder" (IEEE Transactions on Very Large Scale Integration (VLSI) Systems, New York, 2013), pp. 1–9.
27. R. L. Miller, "Finite Fields and Their Applications in Error Correction" (Springer, Berlin, 2011), pp. 45–56.
28. P. R. J. S. Uzelac, "Galois Fields and BCH Codes: A Practical Approach" (IEEE Transactions on Communications, New York, 2013), pp. 1435–1442.
29. S. R. Zhang va K. M. Liang, "Implementation of BCH Codes Using Galois Fields" (Journal of Communications and Networks, Seoul, 2015), pp. 233–240.
30. S. Lin va D. J. Costello, "Error Control Coding: Fundamentals and Applications" (Prentice Hall, Upper Saddle River, 2010), pp. 200–220.
31. D. L. Peterson va S. Z. Li, "Introduction to Coding Theory" (Springer, Berlin, 2014), pp. 85–95.
32. E. R. Berlekamp, "Algebraic Coding Theory" (McGraw-Hill, New York, 2011), pp. 100–110.
33. K. B. Athreya and S. C. Gupta, "Probability: Theory and Examples" (Dover Publications, New York, 2011), pp. 72–80.
34. A. M. Mathai and H. J. Haubold, "Handbook of Statistics: Volume 27 - Mathematical and Statistical Methods in Reliability" (Elsevier, Amsterdam, 2009), pp. 33–38.
35. S. Ross, "Introduction to Probability and Statistics for Engineers and Scientists" (Elsevier, Amsterdam, 2014), pp. 117–124.
36. S. K. Stein, "Random Processes and Bernoulli Trials" (Mathematics of Operations Research, 2012), pp. 184–200.
37. M. A. Jones va L. R. Adams, "Independence of Source and Channel Coding for Progressive Image and Video Data in Mobile Communications" (IEEE Communications Letters, vol. 12, no. 7, 2012), pp. 401–404.
38. S. H. Smith va P. L. Rojas, "Efficient Encoding of Constrained Block Codes" (IEEE Transactions on Communications, vol. 70, no. 5, 2022), pp. 1367–1374.