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THE TRAVELING SALESMAN PROBLEM: MATHEMATICAL MODELING AND OPTIMAL SOLUTIONS

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Abstract: Salesman issues trade in the field of products distribution , sales and transportation costs to calculate These issues trade representative planned routes and work efficiency increase for important is considered . Within the scope of the issue salesman work time , every one the store product sell opportunities , and various to cities to go time determination through total sold products or done affairs This type is issues not only trade representatives efficiency increase , maybe the company general business strategies to optimize help gives .

Key words :distances matrix , route , closed route , route length , whole valuable variables , graph , Hamilton outline (Hamilton cycle), networks and borders method , cited matrix , backpack (bomber aircraft (issue about) , resources , cargo, loads transportation (flight).

Introduction . Processes research and optimal management – decision acceptance to do and systems to optimize scientific fields oriented .

1-Process research resources effective distribution for mathematician models , linear programming , games theory and networks optimization such as from methods uses .

2-Optimal management systems the most good management strategies determination with He is engaged in his work . main methods Pontryagin's Maximum principle and Bellman's dynamic programming .

Literature analysis

Salesman issue and his/her solution according to many scientific research , articles and books They exist . most issue solution for various mathematician and algorithmic approaches offer does. Dantzig , FG, Fulkerson, RW, & Johnson, S. (1954). "Solution of a large-scale traveling-salesman problem." This The work is about the TSP problem . historical beginning Danzig and his/her colleagues this issue mathematician optimization problem as seeing came out and him/her solution for special algorithms create necessity They emphasized . method through the issue of multiple cities for solved . Keller, DM, & Matheson, JE (1972). "The Traveling Salesman Problem: A Survey." This article TSP in solution used various kind approaches , that including heuristic and algorithmic methods about in detail information gives . TSP solution for used solutions efficiency analysis did Applegate, D., Bixby, R., Chvatal , V., & Cook, W. (2006). "The Traveling Salesman

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Problem: A Computational Study." In this book TSP solution for modern computer algorithms application in detail statement done .

Research methodology

Salesman The Traveling Salesman Problem is one or one how many the city visit ordered , every one the city only one times pilgrimage so , the most short the way find The research is methodology this issue solution for various mathematician methods and algorithms using The problem is solved . mathematician Expression : Problem graph to the theory is based on . Each the city as a knot , their between distances and to look at as edges This problem is linear . programming and other mathematician methods is modeled using the heuristic Methods : Heuristic methods issue fast to solve help These methods every always the optimal solution even if it doesn't , it's fast and effective results For example , the Greedy algorithm or Simulation with to add methods used . Computer Simulation : In research computer programs and algorithms using issue solution practice Python or other programming TSP algorithms in languages used . Experimental Analysis : At this stage various cities and roads for issues is created and their The effectiveness of the algorithms is tested . in practice how results to give from the test Analysis and Compare : In the study various algorithms and methods This is done using which method the most effective that determined.To practice Application : Research in the end , found solutions real world to the conditions applies and working release or logistics in the processes how benefit to take possibility is displayed .

Analyses and results

1. Salesman There are **cities .** n Each city the rest transportation route with with connected . Cities between distances matrix $C=(c_{ij}),\quad i,j=\overline{1,n}$ given ($c_{ii}=\quad ,\quad i=\overline{1,n}$ we assume). Salesman somehow from the city out every one in the city only one once become initial to the city return need . To the cities to go route so choice must route Let the length be minimal . This problem mathematician model to compose for

$$x_{ij}=\begin{cases}1, & agar\quad i-\text{shahardan}\quad j-\text{shaharga}\quad \text{borilsa},\\ 0, & \text{aks}\quad \text{holda}.\end{cases} \qquad (1)$$

such as we define this on the ground The issue $i,j=1,2,...,n,\quad i\quad j$ is as follows . is written : (1) by formula determined x_{ij} of variables so values find must ,

$$\sum_{i=1}^{n} x_{ij}=1,\quad j=\overline{1,n}, \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij}=1,\quad i=\overline{1,n}, \qquad (3)$$

$$u_i-u_j+nx_{ij}\quad n-1,\quad i,j=\overline{1,n},\quad i\quad j, \qquad (4)$$

conditions be done and in this

$$f(x)=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij} \qquad (5)$$

function to a minimum achieve .

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(4) in condition u_i variables desired values acceptance they do possible , but commonality without breaking them negative whole we consider it valuable . (2) condition salesman every one to the city only one times entrance possible if (3) is the condition his/her every one from the city only one times exit possible means . (4) condition n cities own inside recipient of the route closed and x_{ii} 1 that provides .

(4) condition when done cities number n from the body small was closed route (cycle) available It won't . Indeed , k if $(k < n)$ the city binder closed route assuming that exists if we do , this route according to (4) condition

$$\sum_{i=1}^{n} u_i - \sum_{j=1}^{n} u_j + nk \quad (n-1)k \qquad (6)$$

come comes out . $\sum_{i=1}^{n} u_i = \sum_{j=1}^{n} u_j$ happened for (6) from $nk \quad (n-1)k, \quad k \quad 0$ relationship we get , that is This is $n \quad n-1$ a contradiction . said the idea Finally, condition (4) is satisfied . satisfactory u_i numbers every one closed route for the existence we show . Indeed , if $x_{ij} = 0$ if (i.e. i from the city j to the city if not possible) (4) condition $u_i - u_j \quad n-1$ appearance takes , this and u_i of voluntary because of is executed . If any k At the -th step i from the city j to the city if it goes , that is $x_{ij} = 1$ if , u_i of voluntary because of , $u_i = k, \quad u_j = k+1$ we can say . At that time $u_i - u_j + nx_{ij} = k - (k+1) + n = n-1$, i.e. condition (4) equality become is done .

The problem is simple. mathematician to the model has although him/her solution to do one row to difficulties Of course , this issue solution for there is was all routes found , their lengths if found out enough . But cities number growth with routes the number is also fast grows . Present in the period salesman about issue solution to do many methods These methods are available . inside networks and borders method his/her own efficiency with separated Below this the method statement we will .

Cities $1,2,3,\ldots,n$ numbers with number them count The ends of the roads and count We consider the arcs of the graph as all from the ends only one once $\mu = \{(i_1,i_2),(i_2,i_3),\ldots,(i_n,i_1)\}$ to the outline Hamilton outline or is called a cycle . Therefore , the problem under consideration is to find the minimum length in the graph . has Hamilton outline from finding consists of .

If $c_{ij} - (i,j)$ arc length if it represents μ cycle length

$$Z(\mu) = \sum_{(i,j) \quad \mu} c_{ij} \qquad (7)$$

will be . μ to cycle in the sum of (7) $C = (c_{ij})$ matrix every line and Only one element from each column is involved . Networks and borders method for route length $Z(\mu)$ of lower border determination important importance has .

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2. Cited matrix and to bring process concepts We enter . If C matrix any i – line or j - column from the elements this of elements the most the youngest let's separate so matrix harvest We will do it . His every one line and on the column no unless one zero there is will be . Harvest was matrix cited matrix , it harvest to do and to bring We call this process . in the process separable elements to the sum bringer immutable it is said and h^k It is said that this on the ground k - to bring process order number .

Bring process more detailed statement Let's do it . Let $c_{i,j(i)} = \min_j c_{ij}, \quad j = 1,2,...,n$ it be . At that time

$$c'_{ij} = c_{ij} - c_{i,j(i)} \qquad (8)$$

will be . $c^*_{i(j),j} = \min_i c'_{ij}, \quad i = 1,2,...,n$ Let it be . At that time

$$c''_{ij} = c'_{ij} - c^*_{i(j),j} . \qquad (9)$$

As a result $C = (c_{ij})$ from the matrix $C = (c''_{ij})$ cited matrix harvest we do . In this bringer immutable

$$h = \sum_{i=1}^{n} c_{i,j(i)} + \sum_{j=1}^{n} c^*_{i(j),j} \qquad (10)$$

from the sum consists of will be . If C matrix C matrix with If we substitute (7) by a similar formula $z(\mu)$ the sum harvest we do . As a result

$$z(\mu) = z'(\mu) + h' \qquad (11)$$

the formula we can , this on the ground h According to (10) is determined . $z'(\mu)$ 0 happened for (11) from $z(\mu)$ h' what harvest we do , that is h' the length of the route (cycle) lower border be takes .

Networks and borders method main the idea We will bring . Initially all Hamilton contours package R for salesman route length lower border $\varphi(R)$ This is determined by the following limit (10) by formula is . Then R collection two to the collection is separated . First collection so Hamilton from the contours consists of , they somehow (i, j) bow own inside This set R_{ij} We denote it as . The second collection (i, j) bow own inside did not receive Hamilton contours package will be . $\overline{R_{ij}}$ We denote it as . Each R_{ij} and $\overline{R_{ij}}$ sets for Hamilton contours lengths lower borders $\varphi(R_{ij})$ and $\varphi(\overline{R_{ij}})$ is determined . Each new lower border $\varphi(R)$ smaller than not . That is R_{ij} and $\overline{R_{ij}}$ from the collections small lower borderline collection is selected and this collection again two to the collection separated and process will be returned . Next harvest was Hamilton is the only one in the collection . to the contour has until this process continue It is delivered . as follows tree in appearance to describe possible .

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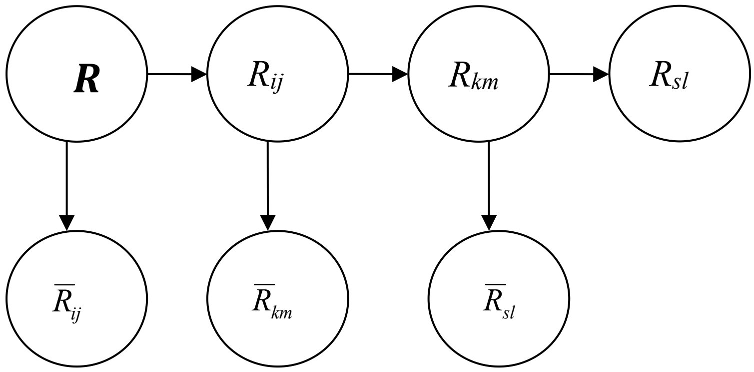

1- rasm

$$\varphi(R) \quad \varphi(R_{ij}) \quad \varphi(R_{km}) \quad \varphi(R_{sl}); \quad \varphi(\overline{R_{ij}}) > \varphi(R_{ij}), \quad \varphi(\overline{R_{km}}) > \varphi(R_{km}), \quad \varphi(\overline{R_{sl}}) > \varphi(R_{sl}).$$

The method algorithm is as follows :

1. $k=1$ that we will get
2. C matrix for C^k cited matrix We'll see .
3. Bringer immutable sum $h^{(k)}$ what We count . He R for lower border will be , that is $\varphi(R)=h^{(k)}$.
4. R_{ij} to the collection to be included the applicants we define , that is so (i,j), $i=1,2,...,n$, $j=1,2,...,n$, i j numbers we find $C_{ij}^k=0$ Let it be .
5. Separated (i,j) for the $\theta^k(i,j)=\min_{j \ j} C_{i,j}^k + \min_{i \ i} C_{i,j}^k$ numbers let's count .
6. $\theta^k(m,l)=\max_{i,j} \theta^k(i,j)$, $C_{ij}^{(k)}=0$ by formula (m,l) the couple we will find and (m,l) bow keeper Hamilton outline package R_{me} what and (m,l) what unsaved Hamilton contours package $\overline{R}_{me}$ what we will find .
7. $\overline{R}_{me}$ collection for lower the border We define this limit . $\varphi(\overline{R_{me}})=\varphi(R)+\theta^k(m,l)$ will be .
8. C^k from the matrix $m-$ line and $l-$ the column we will erase and l to move from m to we prohibit , that is $C_{lm}^k=$ we can say .
9. Harvest was shrunken matrix somehow in step $2 \quad 2$ measurable from the matrix consists of become left and of cities only two possible was couple in itself These pairs using closed route Hamilton contours harvest to do possible . If the shortened matrix $2 \quad 2$ matrix if so, refer to point 11 to point 10 without Let's go .
If the harvest made matrix cited matrix if , R_{me} of the collection lower border this collection harvest made R of the collection lower to the border equal , that is $\varphi(R_{me})=\varphi(R)$. Opposite without harvest made from the matrix cited matrix

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harvest will be done and $h^{(k+1)}$ is considered , $\varphi(R_{me}) = \varphi(R) + h^{(k+1)}$ is found , $k = k+1$ it goes to point 4 .

10. Hamilton contours from taken after branching of the tree disconnected networks is considered and their lower borders outline with length (Record - R_{ec}) compared .

If disconnected to networks suitable sets lower borders R_{ec} smaller than if so , this networks above rule according to continue This process is new harvest was sets lower borders R_{ec} smaller than until continue Networks continue to hold on time new Hamilton contours harvest to be possible . In this case R_{ec} as Hamilton contours length the most small happened is taken .

If the length networks lower borders R_{ec} smaller than Otherwise, the issue is resolved. The optimal route of a traveling salesman is as the most small to length has was route is taken .

Issue

Salesman visits 7 cities (A, B, C, D, E, F, G) order Every to the city only one times Go ahead and start . to the city return The goal is general . distance minimize . Cities between distances matrix as follows given :

	A	B	C	D	E	F	G
A	–	12	10	19	8	11	14
B	12	–	13	7	9	15	10
C	10	13	–	20	6	18	12
D	19	7	20	–	17	9	11
E	8	9	6	17	–	16	7
F	11	15	18	9	16	–	13
G	14	10	12	11	7	13	–

Step 1: Matrix to bring (Line according to)

Each row according to the most small element define it row from the elements We subtract . This is the lower to calculate the limit (H) help gives .

New matrix

	A	B	C	D	E	F	G
A	–	4	2	11	0	3	6
B	5	–	6	0	2	8	3

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C	4	7	–	14	0	12	6
D	12	0	13	–	10	2	4
E	2	3	0	11	–	10	1
F	2	6	9	0	7	–	4
G	7	3	5	4	0	6	–

Lower limit (H):
H = 8 + 7 + 6 + 7 + 6 + 9 + 7 = 50

Step 2: Columns to bring

Each column according to the most small the element let's divide :

New matrix

	A	B	C	D	E	F	G
A	–	4	2	11	0	1	5
B	3	–	6	0	2	6	2
C	2	7	–	14	0	10	5
D	10	0	13	–	10	0	3
E	0	3	0	11	–	8	0
F	0	6	9	0	7	–	3
G	5	3	5	4	0	4	–

Updated H:
H = 50 + (2 + 0 + 0 + 0 + 0 + 2 + 1) = 50 + 5 = 55

Step 3: Results calculation

Each zero for row and column according to the most small values take let's add let's count : line and on the column other the most small values sum

The most big result : 3 (D, B or G, E). (D, B) we choose .

Step 4:

D → B path if selected :

Row D and column B It will be removed .

B → D is forbidden (∞ is set).

Abbreviated matrix (5x5)

	A	C	E	F	G

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A	–	2	0	1	5
C	2	–	0	10	5
E	0	0	–	8	0
F	0	9	7	–	3
G	5	5	0	4	–

E → C is chosen , the matrix again This process continue is delivered , but calculations uncomplicated for complete the route try we see : Route : A → E → C → G → F → D → B → A

Distance : 8 (A→E) + 6 (E→C) + 12 (C→G) + 13 (G→F) + 9 (F→D) + 7 (D→B) + 12 (B→A) = 67

H = 55 (previous border).

D → B path if not selected :

(D, B) = ∞ is set .

New matrix

	A	B	C	D	E	F	G
A	–	4	2	11	0	1	5
B	3	–	6	0	2	6	2
C	2	7	–	14	0	10	5
D	10	∞	13	–	10	0	3
E	0	3	0	11	–	8	0
F	0	6	9	0	7	–	3
G	5	3	5	4	0	4	–

F → D is chosen , the matrix This way longer route to give possible (for example , 70+).

Step 5: Optimal route test

Above the process continue to hold instead of , one how many possible was routes try let's see :

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A → E → G → B → D → F → C → A:
Distance : 8 (A→E) + 7 (E→G) + 10 (G→B) + 7 (B→D) + 9 (D→F) + 18 (F→C) + 10 (C→A) = 69
A → E → C → G → F → D → B → A:
Distance : 67 (above calculated).

Networking during lower limits (H) less than 67 was route not found , so for the most good route as 67 we will get

Answer : The most short route : **A → E → C → G → F → D → B → A** , length **67** .

Conclusion

Salesman optimization problem (TSP) in the field the most important and wide learned from issues This is one of them . The issue in the article is graph. theory , linear programming and heuristic algorithms using mathematician in terms of modeled and his/her solution for networks and borders method such as effective methods seeing was released . Research results this shows that cities number increase The complexity of the issue with exponential accordingly increases , but modern computer algorithms and heuristic approaches real world using under the circumstances satisfactory solutions find possible . The issue practical importance logistics , transportation and business processes in optimization obvious manifestation to be , to be various in the fields application opportunities in the future further expansion is expected . With this together , backpack issue such as additional optimization issues with integration TSP further complicated forms research to do for new opportunities opens .

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