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GRAPHICAL METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEMS

Mamatova Zilolakhon Khabibullokhonovna
Fergana state university associate professor ,
pedagogy sciences according to philosophy Doctor of Philosophy (PhD)
E-mail: mamatova.zilolakhon@gmail.com
ORCID ID 0009-0009-9247-3510
Alimamadov Nurmuhammad Alimardon ugli
Fergana State University
Practical mathematics direction
3rd year student, student of group 22-08
Email: alimamadovnurmuhammad02@gmail.com

Abstract: The graphical solution of linear programming problems is based on the geometric analysis of inequalities and the objective function in systems of two variables (n=2). Each inequality represents a half-plane bounded by a straight line, x_j 0 and the conditions represent areas bounded by the coordinate axes. The solution domain of the system, which can have a common solution, is formed as a convex polygon, a single point, or an empty set. The graphical method includes the following steps: constructing graphs of inequalities, determining the solution domain, drawing the objective function vector, finding the extreme point by parallel displacement in the direction of the vector, and calculating the optimal solution. In the example, the system of inequalities is analyzed graphically, and the maximum (3,0) and minimum (0,3) values of the Z function are determined. This method allows you to solve the problem visually and intelligibly, and practically demonstrates the main principles of linear programming.

Keywords: Linear programming, graphical method, system of inequalities, domain of possible solutions, convex polygon, objective function, extremum point, vector direction, optimal solution, half-plane, coordinate axes, maximum value, minimum value.

Literature review

The following sources and their content are considered for the analysis of the literature on solving linear programming problems graphically. Below is a general analysis and main directions, since a specific list of literature is not given. If you provide a specific list of sources, I can deepen the analysis based on them. Classical textbooks (for example, VA Yemelichev, MM Kovalev, LA Petrosyan, etc.) : In classical textbooks on linear programming, the graphical method is presented as the main approach for problems with two variables. In these sources, the system of inequalities, the concept of a convex polygon, and the process of determining the extreme points of the objective function are explained geometrically. As an advantage of the graphical method, its visual clarity is emphasized, but as a limitation, the impossibility of applying it to problems with many variables is noted. The practical significance of the method is shown through examples (for example, resource allocation or production optimization). Modern literature on mathematical optimization (e.g., D. Bertsimas, JN Tsitsiklis) : In modern sources, the graphical method is discussed not as the

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main theory of linear programming, but as a simplified approach used for educational purposes. In this literature, concepts such as the geometric properties of a convex polygon, the direction of the objective function vector, and the location of the optimal solution at the edge points are thoroughly covered. At the same time, cases of the graphical method with bounded and unbounded solution domains are analyzed. Examples are often focused on logistics, economic modeling, and other areas. Uzbek-language sources (e.g., textbooks of Uzbek universities) : Uzbek-language textbooks (e.g., textbooks on mathematical programming of TDTU or NUU) present the graphical method in a way that is understandable for students. They focus on more practical examples and explain step-by-step the process of constructing a system of inequalities graphically, determining the solution domain, and finding the optimal point. These sources often cite economic and technical issues as examples, but theoretical depth may be limited.

Research methodology

Analysis of the effectiveness of solving linear programming problems graphically, its geometric and analytical properties, as well as the study of the possibilities of educational and practical application of the method. The research is focused on assessing the advantages, limitations of the graphical method and its integration with modern technologies. Theoretical analysis : Literature analysis on the basic concepts of linear programming (system of inequalities, convex polygon, objective function). Study of the mathematical foundations of the graphical method, including the theory of convex sets and the properties of extreme points. Comparison of approaches to the graphical method from various sources (classical and modern textbooks, scientific articles). Mathematical modeling : Formulation of linear programming problems in two variables and their solution using the graphical method. Modeling of cases of various solution domains (bounded polygon, unbounded domain, singular point, empty set). Mathematical verification of the process of determining the minimum and maximum values of the objective function.

Introduction. Process research and optimal control are fields of science focused on decision making and optimization of systems. Process research uses methods such as mathematical models, linear programming, game theory, and network optimization to efficiently allocate resources. Optimal control deals with determining the best control strategies for systems. Its main methods Pontryagin's Maximum principle and Bellman's dynamic programming .

Analyses and results

The following linear equations system given Let it be .

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned}
$$

These equations system vector in uniform as follows is written

$$A \cdot X = B.$$

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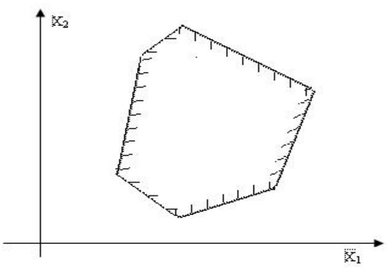
A - equation coefficients matrix ;
X – unknowns vector ;
B - equation free terms vector .
From higher algebra It is known that if n=m and matrix A determinant from scratch different if , that is |A| ≠0 condition if done system to a single solution has will be .

n=2 when inequalities from the system following the system harvest we do :

$$\sum_{j=1}^{2} a_{ij}x_j \le b_i,\ i=\overline{1,m};\quad x_1 \ge 0,\ x_2 \ge 0.$$

Each of these inequalities is a half-plane bounded by a straight line, $a_{i1}x_1 + a_{i2}x_2 = b_i$ and $x_j = 0$ the conditions for the non-negativity of the solutions are half-planes bounded by a straight $x_j \ge 0\ j=1;2$ line . Since the system of inequalities is coupled, it has at least one solution, that is, the boundary straight lines intersect each other, forming a set of possible (reasonable) solutions. Hence , $n=2$ when possible was solutions package polygon from the points consists of will be .



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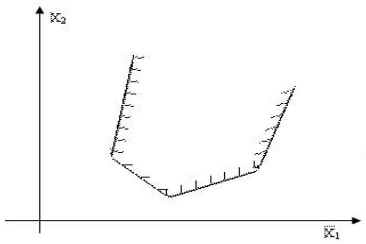

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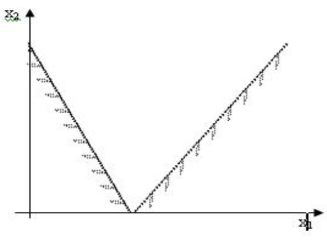

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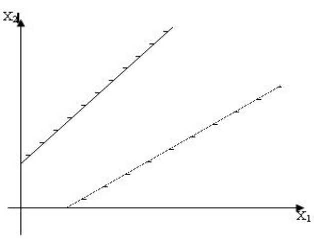

4 pictures

The domain (set) of possible solutions can be a convex polygon (Figure 1), a convex domain with polygons (Figure 2), a singular point (Figure 3), and the empty set (Figure 4).

We write the linear programming problem for two variables as follows.

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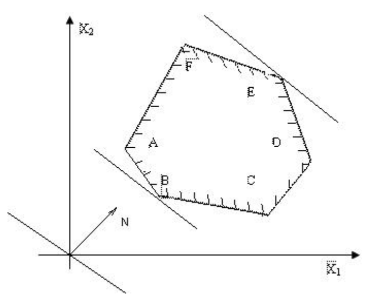
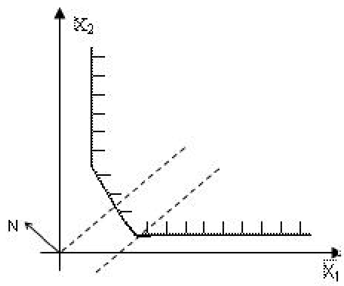
$$a_{11}x_1 + a_{12}x_2 \quad b_1$$
$$a_{21}x_1 + a_{22}x_2 \quad b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 \quad b_m$$
$$x_j \quad 0 \quad (i = \overline{1,m})$$
$$Z = c_1x_1 + c_2x_2 \to \max(\min)$$

Each of the inequalities represents half-planes bounded by lines . A linear function also represents a straight line at a certain constant value. $c_1x_1 + c_2x_2 = const$.

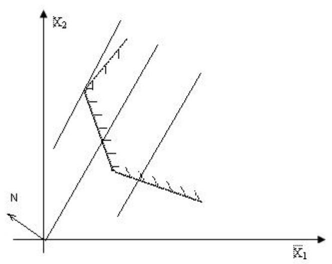
Suppose that the possible solutions consist of a convex polygon. To form a convex set of solutions, we construct a polygon bounded by straight lines. Let this polygon be ABCDEF (Figure 5). The objective function is to give parallel straight lines in the plane X $_1$ OX $_2$. Let the linear function be equal to $c_1x_1 + c_2x_2 = const =$ c_0 an arbitrary constant c $_0$. It forms a straight line. The perpendicular to it is b o ' The vector N(c $_1$,c $_2$) determines the direction of increase of the function Z (Figure 5). If the convex polygon formed by the solutions is not bounded, two cases are possible:

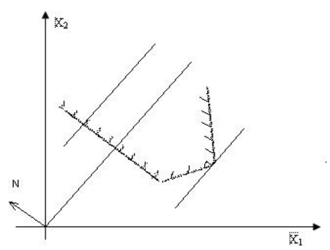
1- take it. $c_1x_1 + c_2x_2 = const$. straight line The vector N(c $_1$,c $_2$) moves along or opposite to it , crossing each and every polygon . However , it does not reach a minimum or maximum value. In this case , the linear function is unbounded from below and above (Figure 6) .

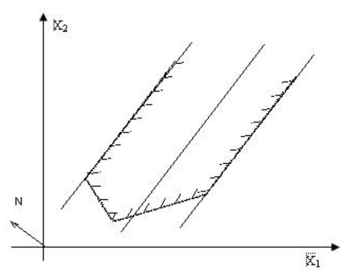




Case 2. $c_1x_1 + c_2x_2 = const$ The straight line N(c $_1$,c $_2$) moves along the vector and reaches a minimum or maximum value at one of the edges of the convex polygon. In this case, the linear function may be bounded above and unbounded below (Figure 3.7) or bounded below and unbounded above (Figure 3.8). Some linear functions limited both above and below to be possible (Figure 3.9).







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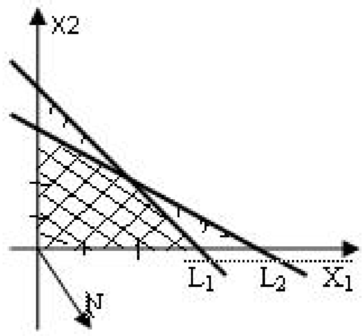
Picture 7 Picture 8 Picture 9

a linear programming problem graphically is done in the following sequence:

1. Graphs of systems of equations or inequalities are constructed .
2. The sides (areas) of each inequality in the plane are determined.
3. The field of possible solutions is separated.
4. The vector N=(c_1,c_2) is constructed and a perpendicular is drawn to it at the point (0,0).
5. The extreme point is found by moving the line parallel to the perpendicular from the polygon in the direction of the vector b . If it is necessary to find the point corresponding to the minimum value of the function Z, then this point corresponds to the first point of the field of possible points when the perpendicular to the vector p is moved in this vector direction. The point giving the maximum value is the last point. If the vector value (negative sign) is -N, the opposite of the above case will happen.
6. The optimal point coordinate is found and the value of the Z function is calculated .

Example. Solve the linear programming problem in the Q -set graphically.

$$4x_1 + 3x_2 \quad 12 \quad (L_1)$$
$$3x_1 + 4x_2 \quad 12 \quad (L_2)$$
$$x_1 \quad 0, \quad x_2 \quad 0,$$
$$Z = 2x_1 - 5x_2 \to \max$$



Given inequalities graphs X_1OX_2 on the plain we build and possible was solutions field (10 pictures) . on the graph cross-hatched place determines . Because this place is all inequalities satisfactory is a field . Possible was solutions the optimal solution from the field Let's determine . Determine from the point (0,0) for the vector N=(2,-5) passing let's make and his/her direction We define . At the point (0,0) this perpendicular to the vector N we will spend and him/her vector direction according to we move . Soha with perpendicular last intersection point to Z function maximum value giver is a point . This point is (3,0) his/her coordinate x1 =3, $x_2 = 0$ of the matter solution will be . From the graph apparently so that Z is a function minimum value and the point giving is (0,3).

Conclusion

The research methodology for solving linear programming problems graphically was aimed at a comprehensive study of the theoretical foundations, practical application and significance of the method in the educational process. The study used theoretical analysis, mathematical modeling, experimental and qualitative-quantitative methods to evaluate the effectiveness of the graphical method in two-variable problems, its geometric properties and integration with modern software tools. The main advantages of the graphical method were its visual clarity and convenience for educational purposes, but its limitations in multivariable problems and problems with accuracy in manual drawing were noted as the main disadvantages. As a result of the study, the practical significance of the graphical method in small-scale problems in the field of economics, logistics and production was confirmed, and at the same time, recommendations were developed for automating the process using tools

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such as MATLAB and GeoGebra. In the future, it is necessary to continue research to expand the application of the method on interactive platforms and ensure its more effective integration into educational programs.

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