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TRANSPORTATION PROBLEM AND ITS SOLUTION METHODS

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Annotation: The transportation problem is an optimization problem in economic mathematics aimed at finding the most cost-effective way to deliver products (resources) from producers to consumers. The main components are: sources (suppliers), demand points (consumers), cost table, and distribution plan. This article also discusses various methods for solving the transportation problem, provides information about Vogel's Approximation Method (VAM), and includes a sample problem solved using the Vogel method.

Keywords: Transportation problem, North-West Corner Method, Potential Method, VAM, Vogel, Python, Excel, resources, demand points, cycles, MODI, Least Cost Method.

Аннотация: Транспортная задача — это задача оптимизации в экономической математике, направленная на нахождение способа доставки продукции (ресурсов) от производителей к потребителям с минимальными затратами. Основными элементами являются: источники (поставщики), пункты потребления (потребители), таблица затрат и план распределения. В данной статье также приведены методы решения транспортной задачи, представлена информация о методе Фогеля и рассмотрен пример решения задачи с использованием метода Фогеля.

Ключевые слова: транспортная задача, северо-западный метод, метод потенциалов, метод Фогеля, VAM, Python, Excel, ресурсы, пункты потребления, циклы, MODI, метод наименьшей стоимости.

Introduction:

The transportation problem is a type of linear programming problem that involves distributing products, services, or resources from producers to consumers at minimal cost. This problem is a significant area within Operations Research and Optimal Control, as it is widely applied in fields such as resource allocation, logistics, supply chains, and production management.

Essence of the transportation problem:

In this problem, there are several supply sources (e.g., factories, warehouses), each with a known quantity of products available. There are also several demand points (e.g., stores, customers), each requiring a specific quantity of products. The cost of transporting goods from each source to each destination is given.The objective is to allocate the products in a way that minimizes the total transportation cost while satisfying all supply and demand constraints.

Main elements:

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1. Sources (Suppliers) – These include factories, plants, or warehouses. They are represented by the available supply of products.
2. Demand Points (Consumers) – These refer to stores, markets, or branches. They are defined by the required quantity of products.
3. Cost Table – This shows the transportation cost per unit for shipping goods from each source to each consumer.
4. Distribution Plan – This indicates the quantity of products to be shipped from each source to each demand point.

Solution methods:

The transportation problem is solved in two main stages.

Methods for creating an initial plan:	Methods for finding the optimal plan:	Solving through computer programs:
North-West Corner Method	MODI – Modified Distribution Method	Using Python
Least Cost Method	Loop method	With Excel: Transportation problems can also be solved using the Excel Solver tool.
VAM -Vogel's approximation method		

Mathematical model of the transportation problem.Let us assume:

- ✓ **m** sources: $A_1, A_2, A_3, A_4 \ldots A_m$
- ✓ **n** demand points: $B_1, B_2, B_3, B_4 \ldots B_n$
- ✓ $\mathbf{a_i}$ is the supply at each source.
- ✓ $\mathbf{b_j}$ is the demand at each consumer point.
- ✓ $\mathbf{c_{ij}}$ is the transportation cost from source **i** to demand point **j**.
- ✓ $\mathbf{x_{ij}}$ is the amount of goods transported from source **i** to demand point **j**.
- ✓ **Z** is the total transportation cost (which should be minimized).
- ✓ **min**- minimize function:

Objective function:

$$Z = \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$$

Objective: To minimize the transportation costs of moving goods or resources from supply points to demand points or maximize profit. To optimally allocate produced products. To reduce transportation time. To make the most efficient use of resources. To organize the logistics system in a cost-effective and efficient manner.

Sample problem: An initial plan is created using Vogel's Approximation Method (VAM). Problem:

<u>3 Warehouses (Sources):</u>

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- A1 – 20 units
- A2 – 30 units

A3 – 25 units

4 Stores (Demand Points):
- B1 – 10 units
- B2 – 25 units
- B3 – 20 units

B4 – 20 units

Transportation costs (in Dollars):

source \ demand	B1	B2	B3	B4
A1	8	6	10	9
A2	9	12	13	7
A3	14	9	16	5

Initial check:
Supply: 20 + 30 + 25 = 75 units
Demand: 10 + 25 + 20 + 20 = 75 units. 75 = 75. Therefore, the problem is balanced, and there is no need to add any artificial supply or demand.
VAM – Vogel's approximation method basics:
1. For each row and column, find the difference between the smallest and the second smallest values (this is the penalty).
2. The row or column with the largest penalty is selected.
3. The maximum possible amount is placed in the cell with the lowest cost in the selected row/column.
4. If supply or demand becomes zero, the corresponding row or column is deleted.

The steps are repeated.

Step 1: Calculating the Differences
For Rows:
- A1: min(6, 8, 9, 10) → 6 and 8 → Difference: 2
- A2: min(7, 9, 12, 13) → 7 and 9 → Difference: 2
- A3: min(5, 9, 14, 16) → 5 and 9 → Difference: 4

For Columns:
- B1: min(8, 9, 14) → 8 and 9 → Difference: 1
- B2: min(6, 9, 12) → 6 and 9 → Difference: 3
- B3: min(10, 13, 16) → 10 and 13 → Difference: 3
- B4: min(5, 7, 9) → 5 and 7 → Difference: 2

Largest Difference: 4 (Row A3)

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1st Selection: Row A3
A3: [14, 9, 16, 5] – Minimum cost: 5 (B4)
Demand (B4): 20 units, Supply (A3): 25 units → We give 20 units from A3 to B4.
We write: x34 = 20, cost: 20 * 5 = 100
New supply (A3): 5 units remaining
New demand (B4): 0 → B4 column is deleted.
Step 2:
We recalculate the differences:

- A1: [8, 6, 10] → 6 and 8 → Difference: 2
- A2: [9, 12, 13] → 9 and 12 → Difference: 3
- A3: [14, 9, 16] → 9 and 14 → Difference: 5

Largest Difference: 5 (Row A3).
Cheapest: 9 (B2)
Columns:

- B1: [8, 9, 14] → Difference: 1
- B2: [6, 9, 12] → Difference: 3
- B3: [10, 13, 16] → Difference: 3

B2 demand: 25, A3 supply: 5 → x32 = 5, cost: 5 * 9 = 45
3rd step:
Largest Difference: A2 (3), Column B2
We look at Row A2 and Column B2: 12
x22 = 20, cost: 20 * 12 = 240
B2 is now completed → deleted
A2: 10 units remaining.
Remaining steps:

- From A1 to B1 (cheapest: 8), x11 = 10, cost: 10 * 8 = 80
- From A2 to B3 (13), x23 = 10, cost: 10 * 13 = 130
- From A1 to B3 (10), x13 = 10, cost: 10 * 10 = 100

A3 is completed and deleted. B2 → 20 units remaining.

Result table:

demand / source	B1	B2	B3	B4
A1	10	-	10	-
A2	-	20	10	-
A3	-	5	-	20

Total cost calculation:

Z = (10×8) + (10×10) + (20×12) + (10×13) + (5×9) + (20×5) = 80 + 100 + 240 + 130 + 45 + 100 = **{695 Dollars }.**

An initial plan was developed using the VAM method. When this plan is checked using the Potential Method (MODI), it is often optimal or near-optimal.

Overall conclusion: The transportation problem is the issue of distributing resources from sources to demand points with the minimum cost. To solve such problems efficiently, various

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algorithms are available, one of which is the Vogel's Approximation Method (VAM). VAM is one of the simple and intuitive methods that provides a near-optimal result for creating an initial feasible plan. This method uses cost differences (penalties) to allocate resources step by step. The practical example above fully demonstrates how the VAM method works, how decisions are made using penalties, how costs are calculated, and how to determine the total cost. As a result, we obtained a near-optimal plan with a total cost of 695 Dollars.

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