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DECISION MAKING UNDER RISK

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Annotation: Decision-making under risk and variable-sum games are significant areas in game theory and decision-making processes. This topic explores the development of optimal strategies in environments characterized by uncertainty and risks. Unlike traditional fixed-sum games, variable-sum games consider scenarios where the total benefit to participants may vary depending on the game's outcome. In these games, participants strive to maximize their own interests, but their decisions depend on the actions of other participants and external uncertainties. Decision-making under risk employs methods such as probability theory, statistical analysis, and scenario modeling. In such conditions, decision-makers must balance expected gains and potential losses based on uncertain information. In variable-sum games, cooperative and competitive strategies play a crucial role, with collaboration or competition among participants influencing the game's outcomes.

Keywords: Risk, variable-sum games, decision-making, uncertainty, game theory, strategic decisions, cooperation, competition.

Introduction

Decision-making under risk and variable-sum games are among the important research areas in modern decision theory and practice. In environments where uncertainty and risks are present, the decision-making processes become complex due to the strategic interactions between participants and their dependence on external factors. Unlike constant-sum games, variable-sum games are characterized by the fact that the total payoff for participants can vary depending on the outcome of the game. In such games, participants aim not only to maximize their own interests but are also compelled to consider the actions of other participants and the uncertain conditions.

Solution Methods:

The following are key methods used in decision-making under risk and uncertainty:

- Hurwicz Criterion
- Expected Value Maximization Method

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- Laplace Criterion
- Minimax–Maximin Methods
- Savage Criterion (Minimax Regret)
- Hoede–Lehmann Method

Objective:

The specific objectives are as follows:

1. To systematically examine the main methods of decision-making under risk, including probability analysis, scenario modeling, and risk management approaches.
2. To analyze the characteristics of variable-sum games, their differences from constant-sum games, and the role of cooperative and competitive strategies.
3. To explore the applicability of core game theory concepts—such as Nash equilibrium, Pareto efficiency, and the Shapley value—to variable-sum games.
4. To evaluate the effectiveness of these methods in solving real-world problems in fields such as economics, management, political science, and artificial intelligence.
5. To develop recommendations for applying the research findings to practical scenarios, such as resource allocation, international negotiations, or business strategies.

Sample Case Study:Problem:

Entrepreneur Sarvar is planning to open a small eatery in the city, specializing in fast food. He is considering three options:

1. A burger shop
2. A shawarma shop
3. A hot dog shop

Depending on market conditions, demand may fall into one of the following three categories:

Demand Level	Probability
High demand	0.3
Moderate demand	0.5
Low demand	0.2

The entrepreneur sells the products at the following prices (**1 USD = 12,000 UZS**):

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Product	**Price (UZS)**	**Price (USD)**
Burger	30,000	$ 2.50
Shawarma	25,000	$ 2.08
Hot dog	18,000	$ 1.50

The estimated monthly sales vary according to each demand level.
Profit matrix for 1 month (in USD):

PRODUCT	**HIGH DEMAND**	**MODERATE DEMAND**	**LOW DEMAND**
Burger	$2,500	$2,000	$750
Shawarma	$1,875	$1,458	$417
Hot dog	$1,500	$1,200	$750

$\beta = 0.7$ $\gamma = 0.3$,

Task:

Which Product to Choose?

Solve the decision problem using the following 6 methods:

1.Hurwicz (Gurvits) Criterion:

This method calculates a weighted average between the best and worst payoffs, where α represents the level of optimism.

2.Expected Value Maximization Method:

This method involves calculating the expected payoff for each option, considering the probabilities of each demand level, and selecting the product with the highest expected value.

3.Laplace Criterion:

Under the assumption of equal probabilities for all possible outcomes, the Laplace criterion calculates the average payoff for each option and selects the one with the highest average.

4.Minimax and Maximin Methods:

These methods focus on minimizing the maximum possible loss (Minimax) or maximizing the minimum gain (Maximin) for each option under different demand scenarios.

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5.Savage (Minimax Regret) Criterion:

This method calculates the regret for each option under each scenario (i.e., the difference between the chosen outcome and the best possible outcome) and selects the product with the minimum maximum regret.

6.Hoede-Lehmann Method:

This method evaluates the overall utility of each option by accounting for both the potential gains and losses under varying demand levels, considering a balanced risk approach.

Solution:

Hurwicz Criterion

We will solve this problem using the Hurwicz criterion. The Hurwicz criterion: 0≤β≤10 The value of β is a parameter, and based on the value of β, the weights wiw_iwi for all i=1,2,...,m are determined.

$$w_i = \beta \times \max \times w_{ij} + (1 - \beta) \times \min \times w_{ij}$$

1.We will solve using the formula.

We will create matrix A.

$$A = \begin{matrix} 2500 & 2000 & 750 \\ 1875 & 1458 & 417 \\ 1500 & 1200 & 750 \end{matrix}$$

Now, based on this, we will find it. $\max \times w_{ij}$

$w_1^* = \max \times w_{1j} = \max(2500,2000,750) = 2500$

$w_2^* = \max \times w_{2j} = \max(1875,1458,417) = 1875$

$w_3^* = \max \times w_{3j} = \max(1500,1200,750) = 1500$

Now, based on this, we will find it. $\min \times w_{ij}$

$w_{1*} = \min \times w_{1j} = \min(2500,2000,750) = 750$

$w_{2*} = \min \times w_{2j} = \min(1875,1458,417) = 417$

$w_{3*} = \min \times w_{3j} = \min(1500,1200,750) = 750$

We will apply the main formula and obtain the result.

$w_1 = \beta \times \max \times w_{ij} + (1 - \beta) \times \min \times w_{ij} == 0.7 * 2500 + (1 - 0.7) * 750 = 1975$

$w_2 = \beta \times \max \times w_{ij} + (1 - \beta) \times \min \times w_{ij} == 0.7 * 1875 + (1 - 0.7) * 417 = 1437{,}6$

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$w_3 = \beta \times \max \times w_{ij} + (1-\beta) \times \min \times w_{ij} == 0.7 * 1500 + (1-0.7) * 750 = 1275$

We will obtain the answer from the results we have.

$\max = w_i = \max(1975, 1437.6, 1275) = 1975$

Thus, it is clear from the results that the answer is w

Answer: The solution α1 should be chosen according to the Hurwicz criterion. Therefore, Sarvar should choose the burger.

2.Maximum Expected Value Method

Let the states of nature be θ1,θ2,...,θn with probabilities p1,p2,...,pn.Then, to find the solution αk we use:

$$w_i = \sum_{j=1}^{n} p_j \times w_{ij}$$

Maximum Expected Value Method $\max w_i$ va α_k.

$w_1 = p_1 \times w_{11} + p_2 \times w_{12} + p_3 \times w_{13} = 0.3 \times 2500 + 0.5 \times 2000 + 0.2 \times 750 = 1900$

$w_2 = p_1 \times w_{21} + p_2 \times w_{22} + p_3 \times w_{23} = 0.3 \times 1875 + 0.5 \times 1458 + 0.2 \times 417 = 1374{,}9$

$w_3 = p_1 \times w_{31} + p_2 \times w_{32} + p_3 \times w_{33} = 0.3 \times 1500 + 0.5 \times 1200 + 0.2 \times 750 = 1200$

Now let's find the $\max w_i$.

$$\max w_i = w_i(1900, 1374.9, 1200) = 1900$$

Therefore, since the $\max w_i = w_1$ solution α_1 should be selected.

Answer: According to the method of maximizing the expected value, Sarvar should choose the Burger.

3. Laplace method

In the Laplace method, the probabilities of the states $\theta_1, \theta_2, \dots, \theta_n$ are assumed to be equal. That is, $p_1 = p_2 = \dots = p_n = \frac{1}{n}$.

$$w_i = \sum_{j=1}^{n} p \times w_{ij} = \frac{1}{n} \times \sum_{j=1}^{n} w_{ij}$$

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Using the formula, we determine the solution α_k.

We find $w_i \ p = \frac{1}{3} \ w_i$

$$w_1 = \frac{1}{n} \times \sum_{j=1}^{n} w_{1j} = \frac{1}{3} \times (2500 + 2000 + 750) = 1750$$

$$w_2 = \frac{1}{n} \times \sum_{j=1}^{n} w_{2j} = \frac{1}{3} \times (1875 + 1458 + 417) = 1250$$

$$w_3 = \frac{1}{n} \times \sum_{j=1}^{n} w_{3j} = \frac{1}{3} \times (1500 + 1200 + 750) = 1150$$

$$\max w_i = \max (1750, 1250, 1150) = 1750$$

So, since $\alpha_k = \alpha_1$, Sarvar should choose the Burger."

Answer: According to the Laplace method, the solution α_1 should be selected. Therefore, the entrepreneur Sarvar should choose the Burger.

4.Minimax and Maximin methods

The solution α_k , which is determined by the minimum of the row-wise maximum values in the θ table, is called the minimax solution.

The solution α_k , which is determined by the maximum of the row-wise minimum values in the θ table, is called the maximin solution.

$$w_1^* = \max \times w_{1j} = \max(2500,2000,750) = 2500$$

$$w_2^* = \max \times w_{2j} = \max(1875,1458,417) = 1875$$

$$w_3^* = \max \times w_{3j} = \max(1500,1200,750) = 1500$$

Now, from this, we determine $\min \times w_i^*$.

$$\alpha_k = \text{minimax}(2500, 1875, 1500) = 1500$$

Therefore, according to the minimax method, $\alpha_k = \alpha_3$, meaning that the Hot-dog should be chosen.

$$w_{*1} = \min \times w_{1j} = \min(2500,2000,750) = 750$$

$$w_{*2} = \min \times w_{2j} = \min(1875,1458,417) = 417$$

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$$w_{*3} = \min \times w_{3j} = \min(1500{,}1200{,}750) = 750$$

Now, from this, we determine $\max \times w_{*i}$.

$$\alpha_k = \text{maxmini}(750, 417, 750) = 750$$

Therefore, according to the maximin method, $\alpha_k = \alpha_1 = \alpha_3$, meaning that both the Burger and the Hot-dog should be chosen.

5.Savage method

In the Savage method, a table called regret (R table) is constructed based on the following rule.

The elements of the R table are

$r_{ij} = \max w_{lj} - w_{ij}$. maxmini α_k By applying the maximin method to the table generated by αk\alpha_kαk , we determine the solution α_k .

$$\max w_{i1} = \max(2500, 1875{,}1500) = 2500$$

$$\max w_{i2} = \max(2000, 1458, 1200) = 2000$$

$$\max w_{i3} = \max(750, 417, 750) = 750$$

We construct the R-table.

$$r_{11} = w_{i1} - w_{11} = 2500 - 2500 = 0$$

$$r_{21} = w_{i1} - w_{21} = 2500 - 1875 = 625$$

$$r_{31} = w_{i1} - w_{31} = 2500 - 1500 = 1000$$

$$r_{12} = w_{i2} - w_{12} = 2000 - 2000 = 0$$

$$r_{22} = w_{i2} - w_{22} = 2000 - 1458 = 542$$

$$r_{32} = w_{i2} - w_{32} = 2000 - 1200 = 800$$

$$r_{13} = w_{i3} - w_{13} = 750 - 750 = 0$$

$$r_{23} = w_{i3} - w_{23} = 750 - 417 = 333$$

$$r_{33} = w_{i3} - w_{33} = 750 - 750 = 0$$

$$R = \begin{matrix} 0 & 0 & 0 \\ 625 & 542 & 333 \\ 1000 & 800 & 0 \end{matrix}$$

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$$r_{*1} = \min(0, 0, 0) = 0$$

$$r_{*2} = \min(625, 542, 333) = 333$$

$$r_{*3} = \min(1000, 800, 0) = 0$$

Therefore, according to the Savage method, α_1 and α_3 should be selected as the solutions.

Answer: According to this method, if the entrepreneur Sarvar chooses the Burger and Hot-dog, his regret will be 0.

6. Hodja-Lemann method

In the Hodja-Lemann method, the parameter $0 \leq \gamma \leq 1$ is involved, and for the probabilities of the states $\theta_1, \theta_2, \dots, \theta_n$, denoted as $p_1, p_2, \dots, p_n$,

$$w_i = \gamma \times p_j \times w_{ij} + (1 - \gamma) \times \min w_{ij}$$

We find the max w_i and α_k using the formula.

$$w_1 = \gamma \times w_1 + (1 - \gamma) \times w_{1*} = 0.3 \times 1900 + 0.7 \times 750 = 1095$$

$$w_2 = \gamma \times w_2 + (1 - \gamma) \times w_{2*} = 0.3 \times 1374.9 + 0.7 \times 417 = 704.37$$

$$w_3 = \gamma \times w_3 + (1 - \gamma) \times w_{3*} = 0.3 \times 1200 + 0.7 \times 750 = 885$$

Therefore, it is clear that the solution is $\alpha_k = \alpha_1$.

Answer: Even after using the Hodja-Lemann method to find the solution, the entrepreneur Sarvar should choose the Burger.

Based on all the answers to this problem, we can conclude that it would be advisable for Sarvar to open a kitchen that prepares burgers. This is because we approached the problem in a real-life context, analyzing it from three different perspectives. In reaching this conclusion, we have reinforced our skills in making decisions under uncertainty, applied theoretical knowledge in practice, and learned to connect it to real-life problems.

Conclusion

Decision-making under uncertainty and variable-sum games are important research areas in modern decision theory and practice, helping to find optimal solutions in environments where uncertainty and strategic interaction exist. This research analyzed the characteristics of variable-sum games, their differences from constant-sum games, and the importance of cooperative and competitive strategies.
In decision-making under uncertainty, methods such as probability analysis, scenario

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modeling, game theory techniques (Nash equilibrium, Pareto efficiency, Shapley value), risk management, and artificial intelligence-based approaches were discussed. The findings of the research show that these methods are crucial in solving real-life problems such as resource allocation, negotiations, and strategic planning in fields such as economics, management, political science, ecology, and artificial intelligence. Variable-sum games allow for balancing cooperation and competition between participants, while risk management methods help make stable and effective decisions in conditions of uncertainty.

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