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MATRIX GAMES-DOMINATION

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Annotatsiya: Ushbu maqolada o‘yinlar nazariyasining asosiy tamoyillari, xususan, ikki shaxsli nol yig‘indi o‘yinlar, bluff strategiyalari va ko‘p bosqichli o‘yinlar tahlil qilinadi. Bellman, Blackwell, Dresher, Ferguson, Kuhn va Nash kabi yetakchi olimlarning ishlari asosida strategik qaror qabul qilishning nazariy va amaliy jihatlari ko‘rib chiqiladi. Optimal strategiyalarni aniqlashda ehtimollik, muvozanat (Gleichgewichtspunkt) va dominatsiya kontseptsiyalariga alohida e’tibor qaratilgan. Tadqiqotlar raqobatbardosh va kooperativ o‘yinlarda optimal natijalarga erishish usullarini ochib beradi.

Kalit so‘zlar: o‘yinlar nazariyasi, nol yig‘indi o‘yinlar, bluff strategiyasi, muvozanat nuqtasi, dominatsiya, ko‘p bosqichli o‘yinlar, strategik qaror qabul qilish.

Annotation: This article analyzes the fundamental principles of game theory, focusing particularly on two-person zero-sum games, bluff strategies, and multi-stage games. Based on the works of leading scholars such as Bellman, Blackwell, Dresher, Ferguson, Kuhn, and Nash, the theoretical and practical aspects of strategic decision-making are examined. Special attention is given to concepts such as probability, equilibrium points (Gleichgewichtspunkt), and domination in identifying optimal strategies. The research reveals methods for achieving optimal outcomes in competitive and cooperative games.

Keywords: game theory, zero-sum games, bluff strategy, equilibrium point, domination, multi-stage games, strategic decision-making.

Аннотация:В данной статье анализируются основные принципы теории игр, в частности, двухличные нулевые игры, стратегии блефа и многоэтапные игры. На основе работ ведущих ученых, таких как Беллман, Блэквелл, Дрешер, Фергюсон, Кун и Нэш, рассматриваются теоретические и практические аспекты принятия стратегических решений. Особое внимание уделяется таким концепциям, как вероятность, точка равновесия (Gleichgewichtspunkt) и доминирование при определении оптимальных

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стратегий. Исследование раскрывает методы достижения оптимальных результатов в конкурентных и кооперативных играх.

Ключевые слова: теория игр, нулевые игры, стратегия блефа, точка равновесия, доминирование, многоэтапные игры, принятие стратегических решений.

A finite two-player zero-sum game in strategic form (X,Y,L), sometimes called a matrix game because the payoff function L can be represented by a matrix. If $X=\{x_1,...,x_m\}$ and $Y=\{y_1,...,y_n\}$ then the game matrix or payoff matrix is expressed as:

$$A=\begin{matrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{matrix}, \quad \text{here } a_{ij}=L(x_i y_j),$$

In this form, player 1 chooses a row, player 2 chooses a column, and player 2 pays player 1 the value in the selected row and column. Note: the values in the matrix are considered as the winnings of the player who chose from the row and the losings of the player who chose from the column.

Mixed strategy for player 1 $p=(p_1,p_2,...,p_m)$ 1in the form of, probabilities that sum to 1is represented as a set of . If player 1uses a mixed strategy and player 2If he chooses the column, then the (average) payoff for player 1 will be:

$$\sum_{i=1}^{m} p_i a_{ij}$$

Also, the mixed strategy of player 2 $q=(q_1,q_2,...,q_n)$ 2will look like this. If player 2strategy and player 1If he chooses the -row, the winnings for player 1 are: $\sum_{j=1}^{n} a_{ij} q_j$

More generally, if player 1strategy and 2nd playerstrategy, then the (average) payoff for player 1 is:

$$p^T A q = \sum_{i=1}^{m} \sum_{j=1}^{n} p_i a_{ij} q_j$$

It is also worth mentioning that Player 1's pure strategy, i.e., only i -row selection, e_i unit vector (only i -position and 0s in the remaining positions). Similarly, player 2's j -column selection e_i is represented by a unit vector.

In the following sections, we will try to "solve" games, that is, find the value and determine at least one optimal strategy for each player. Sometimes we are also interested in determining all optimal strategies.

Balance points. Sometimes the game is easy to solve. If the matrix a_{ij} if it has the following properties:

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1. a_{ij} - i is the smallest element of the array, and
2. a_{ij} - j -if the largest element in the column is

Then to us a_{ij} We call it the equilibrium point. If a_{ij} if the equilibrium point is 1,then player 1 i -select row at least a_{ij} can win the value and player 2 j choose a column and lose a_{ij} may not exceed the value of. Thus, a_{ij} The game will be worth it .

Example 1.

$$A = \begin{matrix} 4 & 1 & -3 \\ 3 & 2 & 5 \\ 0 & 1 & 6 \end{matrix}$$

In this matrix, the middle element 2 is the equilibrium point, since it is the minimum in its row and the maximum in its column. Therefore, it is optimal for player 1 to choose from the second row and for player 2 to choose from the second column. The cost of the game is 2, and the vector (0,1,0) is the optimal mixed strategy for both players.

Large size $m \quad n$ For matrices, checking each element can be tedious. So, as an easier way: calculate the minimum of each row and the maximum of each column and check if there is a match between them.

$$A = \begin{matrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 1 \\ 3 & 1 & 2 & 2 \end{matrix} \quad \text{and} \quad B = \begin{matrix} 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 1 \\ 3 & 2 & 2 & 2 \end{matrix}$$

For matrix A:

- Row minimums: 0,0,0,1
- Column maximums:3,2,2,2

For matrix B:

- Row minimums: 0,0,0,1
- Column maximums: 3,1,2,2

In matrix A, no row minimum is equal to the column maximum, so there is no equilibrium point. However, if in matrix A a_{12} f the element is 1 instead of 2, we get matrix B. In this case, the minimum of the fourth row will be equal to the maximum of the second column. So, b_{42} The inflection point is

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Solution for 2×2 games. Let's generalize Let's consider a 2 × 2 game matrix.

$$A = \begin{matrix} a & b \\ d & c \end{matrix}$$

To solve this game (i.e., find the value of the game and at least one optimal strategy for each player), we proceed as follows:

1. Checking the balance point.
2. If an equilibrium point does not exist, solve by finding equalizing strategies.

Now, we will show how to use the method of equalizing strategies, which works when there is no equilibrium point. Through this method, we can extract the value of the game and the optimal strategies.

Let's assume there is no equilibrium point. If $a \quad b$ so $b < c$, otherwise the equilibrium point will be. $c < b$ because it is not $c > a$ must be , otherwise there will be an equilibrium point again. Continuing like this, we will see that , $d < a$ and $d > b$ should be. In other words ,if $a \quad b$ then $a > b$ and $b < c > d < a$. It can be shown by symmetry that if $a \quad b$ then $a < b > c < d > a$.

This is what I want.shows that: If there is no equilibrium point, then either $a > b, b < c$ and $c > d$ and $d < a$, or $a < b, b > c, c < d$ and $d > a$.

In equations (1), (2) and (3) in Qui, the general 2 2 We will derive optimal strategies and game value formulas for the game. If the player chooses the first row p chooses with probability(i.e., mixedstrategy (p,1− p),), and if player ll uses columns 1 and 2, we equate player l's average utility to:

$$ap + d(1-\mathrm{p}) = \mathrm{bp} + \mathrm{c}(1-\mathrm{p})$$

p If we solve for:

$$p = \frac{c-d}{(\mathrm{a}-\mathrm{b}) + (\mathrm{c}-\mathrm{d})} \qquad (1)$$

Since there is no equilibrium point, $(\mathrm{a}-\mathrm{b})$ and $(\mathrm{c}-\mathrm{d})$ have the same sign (both positive or negative), so $0 < p < 1$. If player ll takes the first column q chooses with probability (i.e.,, $(q, 1-q)$ aralash strategiya ishlatsa), uses a mixed strategy), then we equate the average loss of player ll:

$$aq + b(1-\mathrm{q}) = \mathrm{dq} + \mathrm{c}(1-\mathrm{q})$$

Solving for Q:

$$q = \frac{c-b}{a-b+c-d} \qquad (2)$$

Again, because there is no equilibrium point $0 < q < 1$.

The average loss of player ll under this strategy is:

$$aq + b(1-\mathrm{q}) = \frac{ac-bd}{a-b+c-d} = v$$

This result shows that the game is worthwhile, and both players have optimal strategies (in which case the minimax theorem always guarantees).

Example 2:

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Let's calculate:

q =same as ($q = p = \frac{7}{12}$)

$$v = \frac{8-9}{-2-3-4-3} = \frac{1}{12}$$

Example 3:

$$A = \begin{matrix} 0 & -10 \\ 1 & 2 \end{matrix}$$

$$p = \frac{2-1}{2+10-1-0} = \frac{1}{11}$$

$$q = \frac{2-(-10)}{2+10-1-0} = \frac{12}{11}$$

Here q is greater than 1. What happened? The mistake was that we forgot to check the lower left element (1,1) — it was an equilibrium point. So, there is an equilibrium point in the game.

Removing dominant strategies. Sometimes large matrix games can be reduced in size (hopefully down to 2x2 size) by removing rows and columns that clearly harm the player.

If $A = (a_{ij})$ in the matrix $a_{ij} \quad a_{kj}$ all j For s,then we say, i -string k -takes precedence over -line . If $a_{ij} > a_{kj}$ all j For s,then we say, i -string k -is in a much greater position than the line. Similarly, if $a_{ij} \quad a_{kj}$ (or strictly $a_{ij} > a_{kj}$) all i If it's for you , then j -column k -overrides the column.

Player 1 can achieve any outcome using the dominant row, so dominant rows can be removed. Similarly, dominant columns can be removed. More precisely, removing a dominant row or column does not change the value of the game. However, sometimes the optimal strategy relies on a dominant row or column. If this is the case, then removing that row or column also removes that optimal strategy (but at least one optimal strategy remains). However, if a row or column with a large advantage is removed, the set of optimal strategies does not change.

We can apply this method to multiple rows and columns in a row. For example, consider the following matrix A:

$$A = \begin{matrix} 2 & 0 & 4 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{matrix}$$

We remove the last column and get:

$$\begin{matrix} 2 & 0 \\ 1 & 2 \\ 4 & 1 \end{matrix}$$

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Now the top row is superimposed by the bottom row. We remove the top row and get:

$$\begin{matrix} 1 & 2 \\ 4 & 1 \end{matrix}$$

This 2 2 There is no equilibrium point in the matrix. Therefore, we find mixed strategies using the previous method: $p=\frac{3}{4}, q=\frac{1}{4}$ $v=\frac{7}{4}$. So the optimal strategy in the original game is

For l: $0, \frac{3}{4}, \frac{1}{4}$,

For ll: $\frac{1}{4}, \frac{3}{4}, 0$

It is also possible to remove column strategies through mixed combinations. That is, if a row or column is dominated by a possible combination of several rows, it will also be removed.

For 0<p<1:

$$pa_{ij} + (1-\mathrm{p})\,\mathrm{a}_{i_2 j} \quad a_{kj}$$

if, k -string i_1 and i_2 will be dominated by a mixture of rows. (The same applies to columns.)

Example 4.

$$A = \begin{matrix} 0 & 4 & 6 \\ 5 & 7 & 4 \\ 9 & 6 & 3 \end{matrix}$$

The middle column is a mixture of outer columns (each $\frac{1}{2}$) is preferred over . We remove it.

Then the middle row was dominated by a mixture of the upper and lower rows ($\frac{1}{3}$ and $\frac{2}{3}$). We will remove it too.

It remains:

$$\begin{matrix} 0 & 6 \\ 9 & 3 \end{matrix}$$

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Game value:

$$V = \frac{54}{12} = \frac{9}{2}$$

Conclusion.

It is not always possible to eliminate dominant strategies. If there is an equilibrium point in the matrix, then the game cannot be simplified.3 x 3 Even if there is a balance point in the game, it is still necessary to check the advantage.

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