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SOLVING ECONOMIC PROBLEMS USING DIFFERENTIAL EQUATIONS

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Abstract: This article highlights the role and importance of differential equations in modeling modern economic processes. It analyzes time-dependent economic indicators such as population growth, price dynamics, investment efficiency, and changes in production volume using mathematical models. The application of first-order differential equations is demonstrated using real examples such as the Solow economic growth model, the Malthus population model, and the price equilibrium model. In addition, the process of accumulating interest on a bank deposit savings strategy is modeled using a differential equation. The graphs presented in the article visually illustrate the changes in economic indicators over time. At the end of the study, the use of differential equations as an effective tool in economic analysis and forecasting is emphasized.

Keywords: differential equation, economic modeling, Solow model, Malthus model, price dynamics, population growth, capital changes, linear differential equation, economic forecast, mathematical model.

Time-varying processes are important in the analysis of modern economics. For example, many economic phenomena, such as population growth, investment efficiency, price dynamics, and changes in production, can be transformed into mathematical models using differential equations.

A differential equation is an equation that contains an unknown function and its derivatives**.**

$$\frac{dy(t)}{dt} = f(t, y)$$

here $y(t)-$ economic quantity (e.g. price, population, production volume), $t-$ time.

In economics, these functions are usually time-dependent, for example: price changes, output, population, inflation rate, interest rate, etc.

Solving economic problems using differential equations is a method of analyzing the changes in economic systems and processes over time by mathematically modeling them. This method is used in many economic models and forecasts. Below are the main concepts, areas of application and examples on this topic.

Applications in economic modeling

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Solow economic growth model. (Solow is a famous model of economic growth theory, developed by American economist Robert Solow.)
This model represents output (Y) using capital (K), labor (L), and technology (A). Capital changes are modeled using a differential equation:

$$\frac{dK(t)}{dt} = sY(t) - \delta K(t)$$

here:
$s -$ savings rate,
$\delta -$ capital depreciation ratio,
$Y(t) = A(t)K(t)^{\alpha} L(t)^{1-\alpha}$ – production function (in Cobb-Douglas form).
Population growth model (Malthus model)

$$\frac{dP(t)}{dt} = rP(t)$$

here $P(t) -$ population over time, $r -$ growth rate $t -$ time (in years).
Solution:

$$P(t) = P_0 e^{rt}$$

This model shows exponential population growth.
Example: A city had a population of 100,000 in 2020. The population is growing at a rate of 3% per year. Calculate the population of this city by 2025.
Given: $P_0 = 100000$
$r = 0,03$ (i.e. 3%)
t=5year (2020-2025)
Solution:

$$P(5) = 100000 \ e^{0,03\ 5} = 100000 \ e^{0,15}$$

$$P(5) = 100000 \ 1,1618 = 116180$$

If the growth rate does not change, the population will reach 116,180 in 5 years.
Modeling price changes
Assume that price is time-dependent. Based on supply and demand, the following equation can be constructed:

$$\frac{dp(t)}{dt} = \alpha(D(p) - S(p))$$

here is D(p) the demand function, S(p) the supply function.
Price and supply-demand model: The essence of the problem.
If demand increases, the price increases, and if supply increases, the price decreases. This process needs to be modeled.
Mathematical model:

$$\frac{dp}{dt} = \alpha(D(p) - S(p))$$

Let's assume: $D(p) = a - bp$ (demand), $S(p) = c + dp$ (offer).
Then:

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$$\frac{dp}{dt} = \alpha\left[(a - bp) - (c + dp)\right] = \alpha\left(a - c - (b + d)p\right)$$

This is a first-order linear differential equation.
Solution:

$$\frac{dp}{dt} + \alpha(b + d)p = \alpha(a - c)$$

Solved by integration:

$$p(t) = Ce^{-\alpha(b+d)t} + \frac{a - c}{b + d}$$

This model shows that the price approaches the equilibrium price over time.

The following example mathematically models a savings strategy based on a differential equation:

A woman wants to save $60,000 for her child's future study abroad in 5 years. She uses a deposit in a bank with an annual interest rate of 8%. The bank's interest is calculated continuously.

The woman also deposits an additional $2,000 into the deposit account each year. That is, the total amount of money in the bank increases not only due to interest, but also due to the amount added annually. What should be the initial amount of money in the bank now (at time) so that the total amount in the account after 5 years is $60,000?

$y(t)$ t the amount of money in the bank account at the time (in dollars),
t time (year),
$r = 0,08$ annual interest rate (the interest rate is expressed as a decimal),
The additional amount to be added each year is 2000.

$\frac{dy}{dt} = 0,08y(t) + 2000$ This equation is a first-order linear differential equation in which the growth is due to two factors, an 8% interest rate on the existing amount, and an external input of $2,000 per year.

Initial condition: $y(5) = 60000$ by solving $y(0) -$ We calculate the amount of money that should be deposited in the bank now. As a result of the calculation, the mother should deposit ≈ $ 36,482.59 in the bank today. Then, if $ 2,000 is added every year and the interest rate is 8%, at the end of 5 years the amount of savings will reach $ 60,000.

Graphic examples
The following graphs show the dynamics of various economic models:
Exponential population growth
Capital decline and equilibrium
Price stabilization
Inflation change graph

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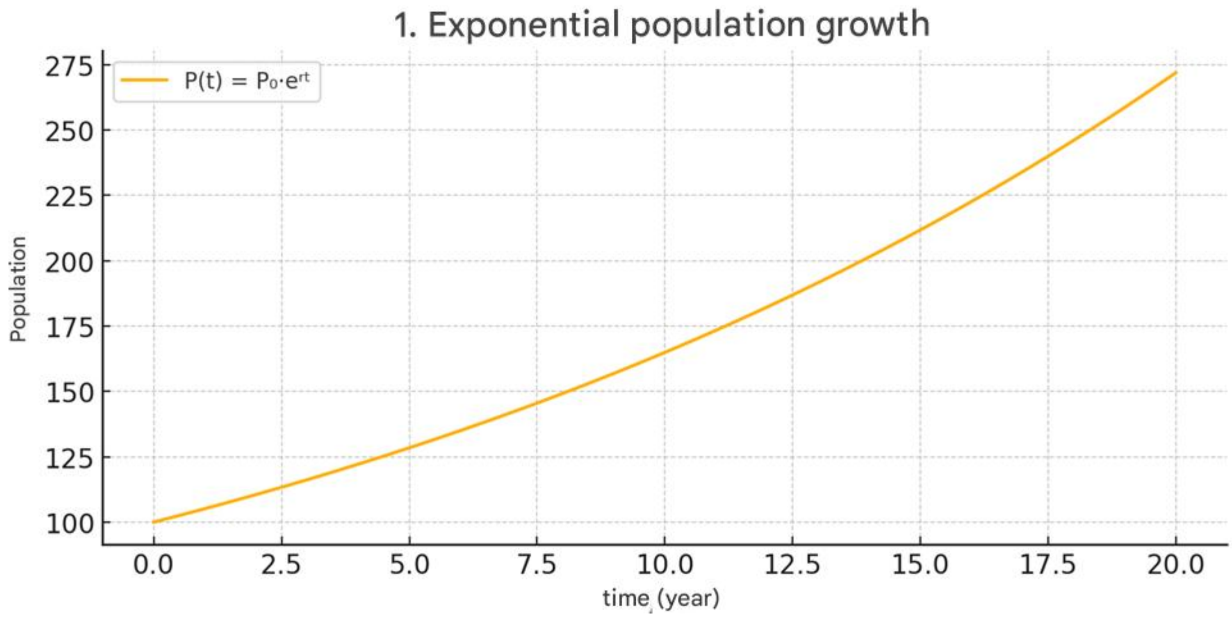

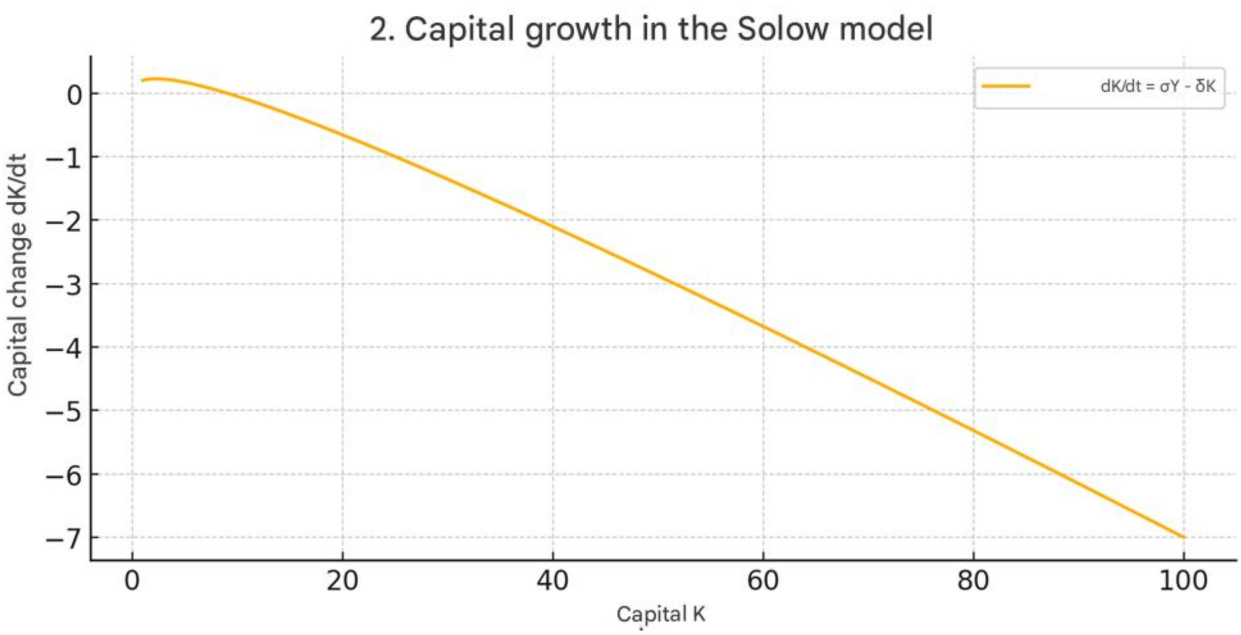

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Conclusion:

Differential equations are a powerful tool for modeling, analyzing, and predicting economic problems. By mathematically representing economic problems, we can gain a deeper understanding of them and make scientifically sound decisions.

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