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DISCRETE VARIANT OF THE INITIAL INTEGRATION METHOD AND ITS APPLICATION TO THE SOLUTION OF BIHARMONIC EQUATIONS

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Let's consider the boundary value problem for the following double harmonic equation:

$$\frac{\partial^4 u}{\partial x^4}+2\frac{\partial^4 u}{\partial x^2 \partial y^2}+\frac{\partial^4 u}{\partial y^4}=-f(x,y), -1<x,y<1, \tag{1}$$

$$u(-1,y)=0, u(1,y)=0,$$
$$u(x,-1)=0, u(x,1)=0.$$

$$\begin{aligned}&\frac{\partial u}{\partial x}(-1,y)=0, \frac{\partial u}{\partial x}(1,y)=0,\\ &\frac{\partial u}{\partial y}(x,-1)=0, \frac{\partial u}{\partial y}(x,1)=0.\end{aligned} \tag{2}$$

Choosing a test function (determining solution) for problems (1)-(2)

$$\begin{aligned}&u_e^{(1)}(x,y)=(1-x^2)^2(1-y^2)^2 e^{A\sin x \sin y},\\ &u_e^{(2)}(x,y)=(1-x^2)^2(1-y^2)^2 e^{A(x+y)}.\end{aligned} \tag{3}$$

To determine the form of the $f(x,y)$ function from formula (1), we substitute solution (3) into equation (1) and will have the following appearance:

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$$
\begin{aligned}
f(x,y) = {} & -e^{A\sin x\sin y}((y^2-1)^2(A\sin y(3(x^2-1)^2\sin^2 x++\sin x(-6A^2(x^2-1)^2 \\
& \sin^2 y\cos^2 x-48Ax(x^2-1)\sin y\cos x+(x^4-74x^2+25))+A\sin y\cos^2 x \\
& (A^2(x^2-1)^2\sin^2 y\cos^2 x+16Ax(x^2-1)\sin y\cos x-4(x^4-20x^2+7))+ \\
& +16x(7-x^2)\cos x)+24)+2\ ((A(y^2-1)^2(x^2-1)(-2A(x^2-1)\cos 2y\cos^2 x- \\
& -\sin x\sin y(x^2-1)+8x\cos x\sin y)+A(y^2-1)^2\sin x\sin y(A(x^2-1)^2\sin y \\
& (-\cos^2 x\sin y+\sin x)+8Ax(1-x^2)\cos x\sin y--12x^2+4)-A(y^2-1)\sin x \\
& \cos y(A\sin x(y^2-1)\cos y+8y)(-A^2(x^2-1)^2\cos^2 x\sin^2 y+A(x^2-1)^2\sin x\sin y- \\
& -8Ax(x^2-1)\cos x\sin y-12x^2+4)-4(3y^2-1)(-A^2(x^2-1)^2\cos^2 x\sin^2 y+ \\
& +A(x^2-1)^2\sin x\sin y-8Ax(x^2-1)\cos x\sin y-12x^2+4)-A(y^2-1)(x^2-1)\cos y \\
& (2A(y^2-1)\sin x\cos y+8y)(-2A(x^2-1)\cos^2 x\sin y+(x^2-1)\sin x-8x\cos x)))+ \\
& +(x^2-1)^2(A\sin x(3(y^2-1)^2\sin^2 y++\sin y(-6A^2(y^2-1)^2\sin^2 x\cos^2 y-48Ay \\
& (y^2-1)\sin x\cos y+(y^4-74y^2+25))+A\sin x\cos^2 y(A^2(y^2-1)^2\sin^2 x\cos^4 y+ \\
& +16Ay(y^2-1)\sin x\cos^3 y-4(y^4-20y^2+7))+16y(7-y^2)\cos y)+24)).
\end{aligned}
$$

To numerically model the differential problem (1)-(2), we first use the discrete variant of the integration method. We search for the approximate solution $u_a(x,y)$ and the right-hand function $f(x,y)$ in the form of a series of finite Chebyshev polynomials of the first kind:

$$
u_a(x,y)=\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}T_i(x)T_j(y),\quad f(x,y)=\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' g_{ij}T_i(x)T_j(y), \tag{4}
$$

here, a_{ij} and g_{ij} are unknowns, and the bar code in the sums means that if the indices i or j are equal to 0, then they are multiplied by $\frac{1}{2}$, and if they are $i=j=0$, then they are multiplied by $\frac{1}{4}$.

In this case, we enter the following lines to calculate the derivatives:

$$
\frac{\partial^4 u_a}{\partial x^4}=\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(4x)}T_i(x)T_j(y),
$$

$$
\frac{\partial^4 u_a}{\partial x^2\partial y^2}=\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(2x,2y)}T_i(x)T_j(y), \tag{5}
$$

$$
\frac{\partial^4 u_a}{\partial y^4}=\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(4y)}T_i(x)T_j(y).
$$

Based on formulas (4)-(5) and equation (1), we form this equation [1-3]:

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$$\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(4x)}T_i(x)T_j(y)+\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(2x,2y)}T_i(x)T_j(y)+\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' a_{ij}^{(4y)}T_i(x)T_j(y)=-\sum_{i=0}^{N}{}'\sum_{j=0}^{M}{}' g_{ij}T_i(x)T_j(y),$$

or by equating the coefficients in front of Chebyshev polynomials of the same order, we form this equation:

$$a_{ij}^{(4x)}+a_{ij}^{(2x,2y)}+a_{ij}^{(4y)}=-g_{ij},$$
$$i=0,1,2,...,N,$$
$$j=0,1,2,...,M.$$

By using a discrete formula of the integration method[1-2]:

$$a_i^{(k-1)x}=\frac{a_{i-1}^{kx}-a_{i+1}^{kx}}{2i}, \qquad (6)$$

both sides of this equation are integrated four times. Then, from the expression of boundary conditions (2) in terms of rows, we form this algebraic system (7):

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$$K_1(j^2-1)((j+1)(j+2)(j+3)a_{i,j-4}+(j-1)(j-2)(j-3)a_{i,j+4})-(K_1(j^2-1)$$
$$(j+1)(j+2)(j+3)+Z_1)a_{i,j-2}-(K_1(j^2-1)(j-1)(j-2)(j-3)+Z_2)a_{i,j+2}+$$
$$+(Z_1+Z_2)a_{i,j}+K_2(j^2-9)(i+1)(j+1)a_{i-2,j-2}-2K_2 j(j^2-9)(i+1)a_{i-2,j}+$$
$$+K_2(j^2-9)(i+1)(j-1)a_{i-2,j+2}+K_2(j^2-9)(i-1)(j+1)a_{i+2,j-2}-2K_2 j$$
$$(j^2-9)(i-1)a_{i+2,j}+K_2(j^2-9)(i-1)(j-1)a_{i+2,j+2}+16j^2(j^2-1)^2(j^2-4)$$
$$(j^2-9)(C_1T_1-C_2T_2+C_3T_3))=-C_1(i-1)j((j+1)(j+2)(j+3)(j^2-1)g_{i-4,j-4}-$$
$$-4(j^2-1)(j^2-4)(j+3)g_{i-4,j-2}+6j(j^2-1)(j^2-9)g_{i-4,j}-4(j^2-1)(j^2-4)$$
$$(j-3)g_{i-4,j+2}+(j-1)(j-2)(j-3)(j^2-1)g_{i-4,j+4})+j(2c_1(i-2)+C_2(i+1))$$
$$((j+1)(j+2)(j+3)(j^2-1)g_{i-2,j-4}-4(j^2-1)(j^2-4)(j+3)g_{i-2,j-2}+$$
$$+6j(j^2-1)(j^2-9)g_{i-2,j}-4(j^2-1)(j^2-4)(j-3)g_{i-2,j+2}+(j-1)(j-2)(j-3)$$
$$(j^2-1)g_{i-2,j+4})-j(C_1(i-3)+2C_2 i+C_3(i+3))((j+1)(j+2)(j+3)(j^2-1)g_{i,j-4}-$$
$$-4(j^2-1)(j^2-4)(j+3)g_{i,j-2}+6j(j^2-1)(j^2-9)g_{i,j}-4(j^2-1)(j^2-4)$$
$$(j-3)g_{i,j+2}+(j-1)(j-2)(j-3)(j^2-1)g_{i,j+4})+j(C_2(i-1)+2C_3(i+2))$$
$$((j+1)(j+2)(j+3)(j^2-1)g_{i+2,j-4}-4(j^2-1)(j^2-4)(j+3)g_{i+2,j-2}+$$
$$+6j(j^2-1)(j^2-9)g_{i+2,j}-4(j^2-1)(j^2-4)(j-3)g_{i+2,j+2}+(j-1)(j-2)(j-3)$$
$$(j^2-1)g_{i+2,j+4})-C_3(i+1)j((j+1)(j+2)(j+3)(j^2-1)g_{i+4,j-4}-4(j^2-1)(j^2-4)$$
$$(j+3)g_{i+4,j-2}+6j(j^2-1)(j^2-9)g_{i+4,j}-4(j^2-1)(j^2-4)(j-3)g_{i+4,j+2}+(j-1)$$
$$(j-2)(j-3)(j^2-1)g_{i+4,j+4}),$$

$$\begin{aligned}
&\frac{1}{2}a_{i,0}+a_{i,2}+a_{i,4}+\ldots+a_{i,2M}=0,\\
&a_{i,1}+a_{i,3}+a_{i,5}+\ldots+a_{i,2M-1}=0, \qquad (i=\overline{0,N}),\\
&4a_{i,2}+16a_{i,4}+36a_{i,6}+\ldots+(2M)^2a_{i,2M}=0,\\
&a_{i,1}+9a_{i,3}+25a_{i,5}+\ldots+(2M-1)^2a_{i,2M-1}=0,\\
&\frac{1}{2}a_{0,j}+a_{2,j}+a_{4,j}+\ldots+a_{2N,j}=0,\\
&a_{1,j}+a_{3,j}+\ldots+a_{2N-1,j}=0, \qquad (j=\overline{4,M}),\\
&4a_{2,j}+16a_{4,j}+36a_{6,j}+\ldots+(2N)^2a_{2N,j}=0,\\
&a_{1,j}+9a_{3,j}+25a_{5,j}+\ldots+(2N-1)^2a_{2N-1,j}=0,
\end{aligned} \tag{7}$$

here

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$$K_1 = 16i^2 j(i^2-1)^2(i^2-4)(i^2-9),$$
$$K_2 = 32ij(i^2-1)(i^2-4)(i^2-9)(j^2-1)(j^2-4),$$
$$T_1 = (i-1)a_{i-4,j} - 2(i-2)a_{i-2,j} + (i-3)a_{i,j},$$
$$T_2 = (i+1)a_{i-2,j} - 2ia_{i,j} + (i-1)a_{i+2,j},$$
$$T_3 = (i+3)a_{i,j} - 2(i+2)a_{i+2,j} + (i+1)a_{i+4,j},$$
$$Z_1 = (3K_1(j-1)(j+2) + 2K_2 i)(j^2-9)(j+1),$$
$$Z_2 = (3K_1(j-2)(j+1) + 2K_2 i)(j^2-9)(j-1),$$
$$C_1 = i(i+1)^2(i+2)(i+3),$$
$$C_2 = 2i(i^2-4)(i^2-9),$$
$$C_3 = i(i-1)^2(i-2)(i-3).$$

In this system, $(N+1)$ $(M+1)$ unknowns $a_{ij}(i=0,1,...,N;\, j=0,1,...,M)$ are determined.

From system (7), the coefficients a_i are obtained and substituted into the formula for the approximate solution:

$$u_a(x,y) = \sum_{i=0}^{N}{}' \sum_{j=0}^{M}{}' a_{ij} T_i(x) T_j(y)$$

The values of the approximate solution at the nodal points (x_l, y_k) are computed and compared with the exact solution $u_a(x,y)$ of the given problem.

The dynamics of the exact and approximate solutions for the function $u_e(x,y)$ are illustrated in Fig. 1 for the case $A=1, N=M=50$.

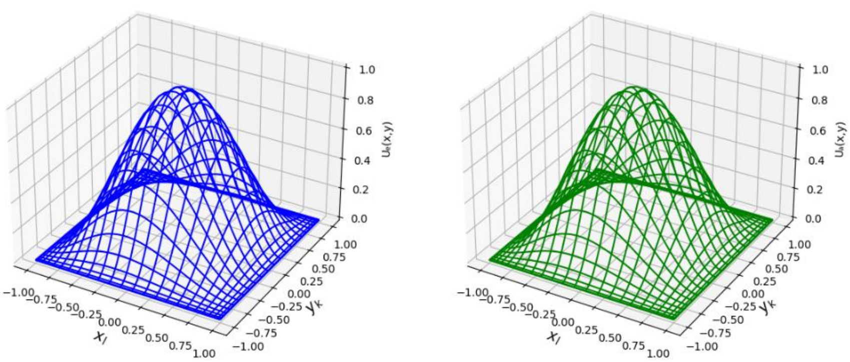

Fig. 1. a) exact solution u_e б) approximate solution u_a

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