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THE PROBABILITY ADDITION FORMULA AND ITS APPLICATION

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Abstract: This article examines the laws of probability theory – formulas of addition and multiplication, their mathematical essence, areas of practical application and examples. These formulas are widely used in statistical analysis, economics, engineering, natural sciences, and information technology.

Keywords: probability theory, probability addition formula, probability multiplication formula, random event, mathematical probability, information technology.

Probability theory is an important branch of mathematics that studies the laws of random phenomena. In practice, there is a need to calculate the probability of events in many issues. One of the most basic concepts in this process is the addition and multiplication formulas. These formulas allow the probability of complex events to be found through the probability of simple events.

The probabilities of a complex event are found by calculating the probabilities of simple events.

The rule for adding the probabilities of disjoint events. The probability of the occurrence of any one of two disjoint events is equal to the sum of the probabilities of these events:

$$P(A+B)=P(A)+P(B)$$

Result: The probability of any one of several events occurring, none of which are mutually exclusive, is equal to the sum of the probabilities of those events.

$$P(A_1+A_2+\ldots+A_n)=P(A_1)+P(A_2)+\ldots+P(A_n)$$

Issue: there are 40 balls in the bowl, 15 of which are white, 5 are green, 20 are yellow. Find the color sphere output probability.

Resolution: color sphere output means that green sphere or yellow sphere output.

A= $\{$ green balloon exit incident $\}$

B= $\{$ yellow balloon exit incident $\}$

Their probabilities, respectively

$$P(A)=\frac{5}{40}=\frac{1}{8} \qquad P(B)=\frac{20}{40}=\frac{1}{2}$$

Events A and B are not shared (one color sphere output omits the other color sphere output).

The probability sought is equal to

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$$P(A+B)=P(A)+P(B)=\frac{1}{8}+\frac{1}{2}=\frac{5}{8}$$

The rule for adding probabilities of shared events. The probability of occurrence of at least one (at least one) of two shared events is equal to the sum of the probabilities of these events subtracting the probability of their shared occurrence:

$$P(A+B)=P(A)+P(B)-P(AB) \qquad (1)$$

This rule can be generalized to any finite number of joint events. for example, for three joint events:

$$P(A+B+C)=P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$$

$n>2$ When this formula takes the following form::

$$P(\sum_{k=1}^{n} A_k)=\sum_{k=1}^{n} P(A_k)-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} P(A_k+A_j)+\sum_{k=1}^{n-2}\sum_{j=k+1}^{n-1}\sum_{s=j+1}^{n} P(A_k+A_j+A_s)-\ldots+(-1)^n P(A_1A_2\ldots A_n)$$

Result. The sum of the probabilities of opposite events is equal to one.:

$$P(A)+P(\overline{A})=1$$

From this $P(\overline{A})=1-P(A)$

Issue. The two hunters fired at the Fox unrelated at the same time. If at least one of the hunters hits the arrow at the target, the Fox will be shot. Find the probability that at least one hunter will hit the target if the first hunter has a hit probability of 0.8 and the second has a hit probability of 0.6.

Solution: Method 1.

A = { first hunter's contact with the target }.

B = { birinchi ovchining nishon bilan aloqasi }.

C=A+B={ at least one hunter's hit }.

Then

$$P(C)=P(A)+P(B)-P(AB)$$

AB both hunters hit the target.

Events A and B are independent events. Therefore

$$P(C)=P(A)+P(B)-P(A)\;\;P(B)=0.8+0.6+0.8\;\;0.6=0.92$$

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Method **2.**

The event of at least one hunter hitting the target and the event of not hitting the target are mutually exclusive events. Therefore

$$P(\overline{A}) = 1 - P(A) = 1 - 0.8 = 0.2 \qquad P(\overline{B}) = 1 - P(B) = 1 - 0.6 = 0.4$$

$$P(C) = P(A+B) = 1 - P(\overline{A})\ P(\overline{B}) = 1 - 0.2\ \ 0.4 = 1 - 0.08 = 0.92$$

Definition. The probability of an event A given that event B has occurred is called the conditional probability of event A given that event B has occurred and is denoted by $P_B(A)$ is defined by.

Definition. If the probability of event A does not depend on whether event b occurred or not, Event A is said to be independent of event B.

Definition. If the probability of event a changes depending on whether event b occurred or not, Event A is said to depend on event B.

The rule for multiplying the probabilities of unrelated events.ition. If the probability of event A does not depend on whether event b occurred or not, their probabilities.: $P(AB) = P(A)\ P(B)$

Result: The probability of several independent events occurring together is equal to the product of the probabilities of these events**.**

$$P(A_1\ A_2 \ldots\ A_n) = P(A_1)\ P(A_2)\ \ldots\ P(A_n)$$

Rule for multiplying the probabilities of dependent events. The probability of two dependent events occurring together is equal to the probability of one of them multiplied by the conditional probability of the other**.**:

$$P(AB) = P(A)\ P_A(B)$$

Result: the probability that several related events occur together equals the probability of one of them multiplied by the conditional probabilities of the rest. At the same time, the probability of each subsequent event is calculated in the hypothesis that all previous events occurred:

$$P(A_1\ A_2\ A_3 \ldots\ A_n) = P(A_1)\ P_{A_1}(A_2)\ P_{A_1 A_2}(A_3) \ldots P_{A_1 A_2 \ldots A_{n-1}}(A_n)$$

here $P_{A_1 A_2 \ldots A_{n-1}}$ - A_n of the event $A_1, A_2, \ldots A_{n-1}$ The probability calculated under the assumption that events occurred.

We can calculate the probability (1) of the occurrence of at least one event by the formula. However, as soon as the number of events is not yet very large, the use of this formula is associated with large computational work. For this reason, another formula is used to calculate this probability.

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Not related together $A_1, A_2, \dots A_n$ The probability of event A occurring, which is the occurrence of at least one of the events

$P(A) = P(A_1 + A_2 + \dots + A_n) = 1 - q_1 q_2 \dots q_n$ is equivalent to.

Here $$\begin{aligned} &p_+ + q = 1, \quad P(A) + P(\overline{A}) = 1 \\ &q = P(\overline{A}_i), \quad i = \overline{1, n} \end{aligned}$$

In particular, $A_1, A_2, \dots A_n$ events p If the probability of each of them is the same, then the probability of at least one of them occurring is

$P(A) = 1 - q^n \qquad (q = 1 - p)$ is equivalent to.

Issue. Each of the three crates has 20 details. The first crease has 10 standard details, the second crease 12, and the third crease 8. One detail per risk is obtained from each crate. Find the probability that all three details obtained will be standard.

Solubility:

A = { The event of obtaining a standard part from the first box }. $P(A) = \dfrac{10}{20} = \dfrac{1}{2}$

B = { The event of a standard part being taken from the second box }. $P(B) = \dfrac{12}{20} = \dfrac{3}{5}$

C={ The event of a standard part being taken from the third box }. $P(C) = \dfrac{8}{20} = \dfrac{2}{5}$

Since events A, B, and C are independent, the probability sought is, according to the multiplication rule,: $P(ABC) = P(A)\ P(B)\ P(C) = \dfrac{1}{2}\,\dfrac{3}{5}\,\dfrac{2}{5} = \dfrac{3}{25} = 0{,}12$ is equal to.

Issue 4. The team has 15 athletes, but 5 of them are masters of sports. 3 athletes are selected by throwing a build from within the athletes. Find the probability that all selected athletes will be masters of sports.

Solution. Method 1**.**

A_1 = { The first athlete to become a master of sports }

A_2 = { The second athlete became a master of sports. }

A_3= { The third athlete became a master of sports. } .

$A = A_1\ A_2\ A_3 =$ { The phenomenon of all three athletes becoming masters of sports} .

$A_1, A_2, \dots A_n$ events are events that are related to each other. So,

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$$P(A) = P(A_1 \ A_2 \ A_3) = P(A_1) \ P_{A_1}(A_2) \ P_{A_1 A_2}(A_3) = \frac{5}{15} \ \frac{4}{14} \ \frac{3}{13} = \frac{2}{21}$$

Method 2.

$A =$ { The phenomenon of all three athletes becoming masters of sports } .

Klassik ta’rifga ko‘ra: $P(A) = \frac{|A|}{|\Omega|} = \frac{C_5^3}{C_{15}^3} = \frac{2}{91}$

An issue concerning the joint application of the rules of addition and multiplication.

Issue. Two snipers are firing at the target. The probability of hitting a target in a single shot is 0.5 for the first sniper and 0.9 for the second sniper. Find the probability that only one of the snipers will hit the target in a single round of fire.

Solutionn issue concerks.

V_1 – only A_1 the incident occurred, that is $B_1 = A_1 \ \overline{A_2}$

V_2 – only A_2 the incident occurred, that is $B_2 = \overline{A_1} \ A_2$

Thus, A_1, A_2 To find the probability of only one of the events occurring, V_1, V_2 The probability of any of the events occurring, regardless of which one, is $P(B_1 + B_2)$ is.

B_1, B_2 Since the events are not together, the addition rule applies. $P(B_1 + B_2) = P(B_1) + P(B_2)$

$$P(B_1) = P(A_1 \overline{A_2}) = P(A_1) P(\overline{A_2}) = p_1 q_2$$

$$P(B_2) = P(\overline{A_1} A_2) = P(\overline{A_1}) P(A_2) = q_1 \ p_2$$

So, $P(B_1 + B_2) = p_1 \ q_2 + q_1 \ p_2 = 0,5 \ 0,1 + 0,5 \ 0,9 = 0,05 + 0,45 = 0,5$

Application of addition and multiplication formulas

1. • In assessing risks in statistics and economics.
2. • When calculating the possibility of occurrence of events in the insurance industry.
   • In Informatics and cryptography.
   • In determining genetic combinations in biology and medicine.

Probability addition and multiplication formulas are one of the main tools in random event analysis. They are widely used not only in theoretical mathematics, but also in everyday life, economics, technology and Natural Sciences. With these formulas, the probability of complex events is calculated through the probability of simple events.

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