ORIGINAL ARTICLE

INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE

ISSN: 2692-5206, Impact Factor: 12,23
American Academic publishers, volume 05, issue 10,2025

Journal: https://www.academicpublishers.org/journals/index.php/ijai

UNUSUAL TYPES OF IRRATIONAL EQUATIONS AND METHODS FOR SOLVING THEM

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Introduction:

This article examines one of the key topics in mathematics that requires critical thinking and contributes to the development of cognitive abilities — **"Irrational Equations."** In particular, it analyzes examples of solving such equations by **raising both sides to a power**, as well as by **multiplying both sides by the conjugate expression**.

The main goal of this article is to encourage students to adopt **innovative ways of thinking** when solving various mathematical problems and to promote the use of **non-standard methods** in the process. Additionally, through the analysis of this topic, the article aims to teach secondary school students how to determine **when irrational equations can be solved** either by raising both sides to a power or by multiplying them by the conjugate expression.

Keywords: irrational equations, domain of definition, introducing a new variable, denoting an expression.

We begin this article with the words of Bertrand Russell (1872–1970), an English philosopher and mathematician:

"The beauty of mathematics lies in showing how deep truths arise from simple ideas."

This is particularly relevant to our article, as here we focus on demonstrating how simple ideas can be applied to solve seemingly complex irrational equations, leading to remarkable results.

Furthermore, in this article, we highlight examples where brilliant ideas can be applied under the topic of “Irrational Equations”, such as “Solving equations by raising to a power” and “Finding solutions by multiplying or dividing by irrational expressions”, and we analyze the methods for solving these examples.

Analysis of Examples Solved by Raising to a Power:
Example 1:

$\sqrt{x+\sqrt{x+11}}+\sqrt{x-\sqrt{x+11}}=4$ “Solve the equation on [2,3]”

Solution: $\sqrt{x+\sqrt{x+11}}+\sqrt{x-\sqrt{x+11}}=4$ To solve the equation, we raise both sides to a power of two, because under the radicals on the left-hand side of the equation, there are expressions that are combined with opposite signs. By squaring, the sum of these oppositely signed terms becomes zero, and their product takes the form of the short multiplication formula. That is,

$$\left(\sqrt{x+\sqrt{x+11}}+\sqrt{x-\sqrt{x+11}}\right)^2=16$$

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$$x+\sqrt{x+11}+2\sqrt{\left(x+\sqrt{x+11}\right)\left(x-\sqrt{x+11}\right)}+x-\sqrt{x+11}=16$$

$$2x+2\sqrt{x^2-x-11}=16$$

$$x+\sqrt{x^2-x-11}=8$$
$$\sqrt{x^2-x-11}=8-x$$

Then, we proceed with this equation just as we did for the equations considered in [1]: first, we determine the domain of definition, and then we raise both sides of the equation to the power of two.

$$\sqrt{x^2-x-11}=8-x \qquad \begin{matrix} 8-x \quad 0 \\ x^2-x-11 \quad 0 \end{matrix}$$

$$x \quad 8$$
$$\left(x-\frac{1-3\sqrt{5}}{2}\right)\left(x-\frac{1+3\sqrt{5}}{2}\right) \quad 0 \qquad x \quad -11;\ \frac{1-3\sqrt{5}}{2} \qquad \frac{1+3\sqrt{5}}{2};\ 8$$

Now, we square both sides of the equation.

$$\left(\sqrt{x^2-x-11}\right)^2=(8-x)^2$$
$$x^2-x-11=x^2-16x+64$$
$$15x=75$$
$$x=5$$

Tenglamaning yechimi uning aniqlanish sohasiga tegishli bo'lganligi uchun, tenglamaning yechimi mavjud va u $x=5$ ga teng ekan.

Misol 2: $\sqrt{x^2+x-5}+\sqrt{x^2+8x-4}=5$ "Solve the equation on [2,4]"

Example 1: $\sqrt{x^2+x-5}+\sqrt{x^2+8x-4}=5$ To find the solution of the equation, we first move one of the radicals from the left-hand side of the equation to the right-hand side, and then we square both sides of the resulting equation.

$$\sqrt{x^2+x-5}=5-\sqrt{x^2+8x-4}$$

$$\left(\sqrt{x^2+x-5}\right)^2=\left(5-\sqrt{x^2+8x-4}\right)^2$$

$$x^2+x-5=25-10\sqrt{x^2+8x-4}+x^2+8x-4$$

$$x-5=21-10\sqrt{x^2+8x-4}+8x$$

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$$7x+26=10\sqrt{x^2+8x-4}$$

$$49x^2+364x+676=100x^2+800x-400$$

$$51x^2+436x-1076=0$$

An equation is obtained, and if we solve this quadratic equation,

$$x_1=2,\ x_2=-\frac{538}{51}\approx-10,5$$

It turns out that both of these solutions may or may not be solutions of the equation, so this needs to be verified.
For this purpose, as mentioned in [1], it is necessary to determine the domain of definition of the equation. However, for some equations, after finding the domain, it is advisable to substitute the obtained solution back into the equation to check its validity. Following the same approach, we check the solutions of the above equation and determine which of them are indeed valid solutions.

Example 3:

$\sqrt[3]{x+34}-\sqrt[3]{x-3}=1$

$$\left(\sqrt[3]{x+34}-\sqrt[3]{x-3}\right)^3=1$$

$$x+34-3\sqrt[3]{(x+34)(x-3)}\left(\sqrt[3]{x+34}-\sqrt[3]{x-3}\right)-x+3=1$$

$$3\sqrt[3]{(x+34)(x-3)}\left(\sqrt[3]{x+34}-\sqrt[3]{x-3}\right)=36$$

$$\sqrt[3]{x+34}-\sqrt[3]{x-3}=1$$

$\sqrt[3]{(x+34)(x-3)}=12$

$x^2+31x-1830=0$ An equation is obtained. The solutions of this quadratic equation are $x_1=-61$ and $x_2=30$ and both of these solutions can be valid solutions of the original equation.

Analysis of Examples for Solving Equations Using the "Multiplication by Sum" Method:

Example 4: $\sqrt{3x^2+5x+8}-\sqrt{3x^2+5x+1}=1$

Yechish: In solving this equation, we use one of the unusual methods called the multiplication by sum method. This allows us to transform the equation into a system of equations of the form $\begin{matrix} x_1+x_2=a \\ x_1-x_2=b \end{matrix}$. Since the unknown parts under both radicals of the equation are the same, multiplying and dividing by their sum eliminates these parts.

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$$\frac{3x^2+5x+8-3x^2-5x-1}{\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}}=1$$

$$\frac{7}{\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}}=1$$

$$\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}=7$$

This results in **[expression/result not specified]**, and if we combine the obtained result with the given example, then

$$\sqrt{3x^2+5x+8}-\sqrt{3x^2+5x+1}=1$$
$$\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}=7$$

A system of equations of the form,

$\begin{matrix}\sqrt{3x^2+5x+8}=4\\ \sqrt{3x^2+5x+1}=3\end{matrix}$ from which follows. From this, we arrive at the following quadratic equation.

$3x^2+5x+8=16$ $\quad 3x^2+5x-8=0$ which is the solution of this equation.

$x_1=1,\quad x_2=-\dfrac{8}{3}$

Conclusion:

Up to the 10th grade, this topic is not included in the mathematics curriculum, and generally, teaching school students the topic in a way that involves irrational expressions and functions on both sides of the equation, as presented above, may lead to a number of misunderstandings. Therefore, in general secondary schools, it is advisable to simplify the topic initially, taking into account that students in each class have different interests and levels of understanding of the material, and then gradually introduce examples and problems of higher difficulty.

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