academic publishers
INTERNATIONAL JOURNAL OF ARTIFICIAL INTELLIGENCE (ISSN: 2692-5206)
Volume 04, Issue 04, 2024
Published Date: - 30-06-2024

SOLUTION OF THE CAUCHY PROBLEM FOR THE MULTIDIMENSIONAL GENERALIZED EULER–POISSON–DARBOUX EQUATION BY THE METHOD OF SPHERICAL MEANS

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Abstract: This article examines the solution of the Cauchy problem for the multidimensional generalized Euler-Poisson-Darboux equation using the method of spherical means. A special approach is employed, based on the expansion of the solution function into a series of spherical harmonics. The article details this approach and proposes an algorithm for its implementation. Numerical experiment results are also presented, demonstrating the effectiveness of the proposed method. Overall, the article constitutes a significant contribution to the study of multidimensional Euler-Poisson-Darboux equations and may be of interest to specialists in mathematics and physics.

Key words: Euler-Poisson-Darboux equation, Cauchy problem, multidimensional generalization, method of spherical means, spherical harmonics, Fourier series, explicit solution, numerical modeling, solution stability, convergence, applications in physics, applications in mechanics, mathematical physics, partial differential equations, solution algorithm.

Introduction

The Euler-Poisson-Darboux multidimensional equations are among the most fundamental equations in mathematical physics. They find wide application in various fields, including hydrodynamics, gravitational physics, and elasticity theory. Solving the Cauchy problem for these equations is an important task, the solution of which allows obtaining information about the behavior of the system in the future based on its initial state.

The method of spherical means is one of the most effective methods for solving the Cauchy problem for the multidimensional generalized Euler-Poisson-Darboux equation. This method is based on the use of spherical functions and spherical means, which allow reducing the multidimensional problem to a one-dimensional one. This significantly simplifies the calculations and reduces the computational complexity of the problem.

This article will discuss the method of spherical means for solving the Cauchy problem for the multidimensional generalized Euler-Poisson-Darboux equation. The main principles of the method, its advantages, and disadvantages will be considered. An example of solving the Cauchy problem for a specific multidimensional Euler-Poisson-Darboux equation will also be discussed.

In addition, the method of spherical means is a powerful tool for solving the Cauchy problem for multidimensional Euler-Poisson-Darboux equations. It significantly simplifies the calculations and reduces the computational complexity of the problem. However, like any method, it has its limitations and disadvantages that must be taken into account when applying it.

Research:

Partial differential equations with singular coefficients have a rich history. For the first time the equation

$$u_{xy} - \frac{\alpha}{x-y}u_x + \frac{\beta}{x-y}u_y + \frac{\gamma}{(x-y)^2}u = 0, \qquad (1)$$

Where α, β, γ - const, obtained by L. Euler [1] in connection with the study of air movement in pipes of different sections and vibrations of strings of variable thickness. He gave a solution to this equation at $\alpha = \beta = m, \gamma = n$, where $m, n \quad N$.

The general solution of equation (1) at $\alpha = \beta$ found by B. Riemann [2], who constructed a solution to the Cauchy problem using an auxiliary function and a method that was later named after him.
An equation like (1), but in the form

$$E^{-}_{q,p}(u) \quad u_{xx} - u_{yy} - \frac{2q}{y}u_x - \frac{2p}{y}u_y = 0, \qquad (2)$$

Where q, p - const, with $q = 0$ solved by S. Poisson [3] by finding for it a hyperbolic analogue of the solution representation, called the Poisson representation. In this work he also considered the equation

$$L_p(u) \quad \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2} - \frac{\partial^2 u}{\partial y^2} - \frac{2p}{y}\frac{\partial u}{\partial y} = 0, \qquad (3)$$

at $n = 3, \ p = 1$.

Much later, equation (2) at $q = 0, 0 < p < 1$ was encountered when studying the issues of surface curvature in the monograph by G. Darboux [4], where it was called the Euler-Poisson equation. Therefore, subsequently, many authors began to call equations of the form (1), (2), (3) and their elliptic analogues Euler-Poisson-Darboux equations.

Interest in such equations increased significantly after the publication in 1923 of the first edition of the book by F. Tricomi [5], where equations of the form (1), (2) and

$$E^{+}_{q,p}(u) \quad u_{xx} + u_{yy} + \frac{2q}{y}u_x + \frac{2p}{y}u_y = 0, \qquad (4)$$

at $q = 0, \ p = 1/6$ significantly used in the study of the boundary value problem for an equation of mixed elliptic-hyperbolic type $yu_{xx} + u_{yy} = 0$, later called the Tricomi equation. Moreover, many equations of mixed type, for example, the generalized Tricomi equation, the Carol equation, a number of equations of mixed type with degeneracy of type and order, etc., in their hyperbolicity regions are reduced to the Euler-Poisson-Darboux equation.
Nevertheless, using a change of variables, it is possible to reduce a fairly wide class of degenerate equations, both of the first and second kind, to equations with singular coefficients. For example, an equation with degeneracy of type and order

$$y^m \quad \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2} - y^k \frac{\partial^2 u}{\partial y^2} - \alpha y^{k-1} \frac{\partial u}{\partial y} - \lambda^2 y^k u = 0 ,$$

by replacement $t = [2/(m-k-2)] y^{\frac{m-k+2}{2}}$ reduce to equation

$$L_p^{\lambda}(u) \quad \sum_{k=1}^{n} \frac{\partial^2 u}{\partial x_k^2} - \frac{\partial^2 u}{\partial t^2} - \frac{2p}{t}\frac{\partial u}{\partial t} - \lambda^2 u = 0 . \qquad (5)$$

More detailed information in this area can be found in the monographs of A.V. Bitsadze [6] and M.M. Smirnov [7-8].

An important role in the creation of the theory of Euler-Poisson-Darboux equations and their analogs was played by the worksA. Weinstein [9-13]. In these articles A. Weinstein for different values of the parameter p studied the Cauchy problem for equation (3) with semihomogeneous initial conditions

$$u(x,0) = \varphi(x),\ u_t(x,0) = 0,\ x \quad R^n \qquad (6)$$

and its solution is obtained explicitly.

It also contains correspondence formulas of the form

$$E_{q,p}^{+}(y^{1-2p}u) = y^{1-2p} E_{q,1-p}^{+}(u) , \qquad (7)$$

for equation (4) at $q = 0,\ 0 < p < 1/2$. Note that a formula like (7) was found in G. Darboux [4].

Cauchy problem with initial conditions

$$u(x,0) = \varphi(x), \quad \lim_{t \to +0} t^{2p} u_t(x,t) = \psi(x),\ x \quad R^n , \qquad (8)$$

for equation (5) at $\lambda \quad 0,\ 0 < p < 1/2,\ n = 1,2$ was studied by M.B. Kapilevich in [14], and when $n = 3$ it was studied in [15].

ECYoung's work [16] contains a review of research on the singular Cauchy problem {(3), (8)}. In the works of JBDiaz, HFWeinberger [17], EK Blum [18], the problem {(3), (8)} was studied for various values of the parameter p.

The uniqueness of the solution to the Cauchy problem {(3), (8)} was proven in the works of EK Blum [19], DW Bresters [20], DW Fox [21]. However, as shown in the work of DW Bresters [20], the solution to this problem in the case $p < 0$ is not the only one.

In this paper, we consider the Cauchy problem for the hyperbolic equation (5), with initial conditions (8) at $0 < p < 1/2$ and $\lambda \quad 0,\ n \quad 1$. To solve this problem, we apply the method of spherical averages [22].

Let $S(x,r) = \{\xi : |\xi - x| = r\}$ - radius sphere r centered at a point $x \quad R^n$, where $|\xi - x|^2 = \sum_{k=1}^{n} (\xi_k - x_k)^2$ distance between points ξ And x. Let, further,

$$U(x,r,t) = \frac{1}{\omega_n r^{n-1}} \int_{S(x,r)} u(\xi,t)\, d\sigma_\xi = \frac{1}{\omega_n} \int_{S(O,1)} u(x + r\eta, t)\, d\omega , \qquad (9)$$

Where $\omega_n = 2\pi^{(n/2)} / \Gamma(n/2)$, $S(O,1) = \{\eta : |\eta| = 1\}$ - unit sphere with center at the origin, $d\sigma_\xi$ - surface element of a sphere $S(x,r)$, $d\omega$ -element of the surface of the unit sphere, and $d\sigma_\xi = r^{n-1} d\omega$, $\Gamma(z)$ - gamma – Euler function.

Obviously, equality (9) is the arithmetic mean value of the function $u(x,t)$ on the sphere $S(x,r)$.

Using the classical method of spherical averages [22], it can be shown that if the function $u(x,t)$ is a solution to the Cauchy problem {(5), (8)}, then the function $U(x,r,t)$ will be a solution to the equation

$$U_{rr}+\frac{n-1}{r}U_r-U_{tt}-\frac{2p}{t}U_t-\lambda^2U=0\quad, \qquad (10)$$

satisfying initial

$$U(x,r,0)=\Phi(x,r), x \quad R^n, r \quad R^1_+, \ \lim_{t\to+0}t^{2p}U_t(x,r,t)=\Psi(x,r), \ x \quad R^n, r \quad R^1_+, \quad (11)$$

and boundary conditions

$$U_r(x,0,t)=0, \ x \quad R^n, t>0\ , \qquad (12)$$

where

$$\Phi(x,r)=\frac{1}{\omega_n r^{n-1}}\ _{S(x,r)}\ \varphi(\xi)d\sigma_\xi=\frac{1}{\omega_n}\ _{S(O,1)}\ \varphi(x+r\eta)d\omega\,, \qquad (13)$$

$$\Psi(x,r)=\frac{1}{\omega_n r^{n-1}}\ _{S(x,r)}\ \psi(\xi)d\sigma_\xi=\frac{1}{\omega_n}\ _{S(O,1)}\ \psi(x+r\eta)d\omega\,, \qquad (14)$$

and $\Phi_r(x,0)=0, \ \Psi_r(x,0)=0$.

Considering equality

$$U_{rr}+\frac{n-1}{r}U_r=r^{-(n-1)}\frac{\partial}{\partial r}\quad r^{n-1}\frac{\partial U}{\partial r}$$

and multiplying equation (10) by r^{n-2}, let's rewrite it in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\quad r^{n-1}\frac{\partial U}{\partial r}\quad-\frac{\partial^2}{\partial t^2}\left(r^{n-2}U\right)-\frac{2p}{t}\frac{\partial}{\partial t}\left(r^{n-2}U\right)-\lambda^2\left(r^{n-2}U\right)=0\,. \qquad (15)$$

Let $n=2k+1$. Applying the differential operator to equation (15) $\quad\frac{1}{r}\frac{\partial}{\partial r}\quad^{k-1}$, we get

$$\frac{1}{r}\frac{\partial}{\partial r}\quad^{k}\quad r^{2k}\frac{\partial U}{\partial r}\quad-\frac{\partial^2}{\partial t^2}\quad\frac{1}{r}\frac{\partial}{\partial r}\quad^{k-1}\left(r^{2k-1}U\right)\quad-$$

$$-\frac{2p}{t}\frac{\partial}{\partial t}\quad\frac{1}{r}\frac{\partial}{\partial r}\quad^{k-1}\left(r^{2k-1}U\right)\quad-\lambda^2\quad\frac{1}{r}\frac{\partial}{\partial r}\quad^{k-1}\left(r^{2k-1}U\right)=0\,. \qquad (16)$$

The following lemma holds.

Lemma 1([23]). If $w(r)\quad C^{k+1}, k=1,2,\ldots$, then the equalities are true

1. $$\frac{d^2}{dr^2}\quad\frac{1}{r}\frac{d}{dr}\quad^{k-1}\left(r^{2k-1}w\right)=\quad\frac{1}{r}\frac{d}{dr}\quad^{k}\quad r^{2k}\frac{dw}{dr}\quad, \qquad (17)$$

2. $$\frac{1}{r}\frac{d}{dr}\quad^{k-1}\left(r^{2k-1}w\right)=\ _{j=0}^{k-1}\ A^k_j r^{j+1}\frac{d^jw}{dr^j}, \qquad (18)$$

where $A^k_j=const$, and $A^k_0=1\ \ 3\ \ 5\ \ 7\cdots(2k-1)=(2k-1)!!$.

This lemma can be proven by mathematical induction.

By entering the designation

$$V(x,r,t)=\quad\frac{1}{r}\frac{\partial}{\partial r}\quad^{k-1}\left(r^{2k-1}U\right), \qquad (19)$$

and taking into account Lemma 1, with respect to the function $V(x,r,t)$ we obtain the problem of finding a solution to the equation

$$V_{rr} - V_{tt} - \frac{2p}{t}V_t - \lambda^2 V = 0, \qquad (20)$$

satisfying initial

$$V(x,r,0) = f(x,r),\ x \in R^n,\ r > 0,\ \lim_{t\to+0} t^{2p}V_t(x,r,t) = g(x,r),\ x \in R^n,\ r > 0 \qquad (21)$$

and boundary conditions

$$V(x,0,t) = 0,\ x \in R^n,\ t > 0, \qquad (22)$$

where

$$f(x,r) = \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{k-1}\left(r^{2k-1}\Phi(x,r)\right), \qquad (23)$$

$$g(x,r) = \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{k-1}\left(r^{2k-1}\Psi(x,r)\right). \qquad (24)$$

In [14], problem {(20), (21)} was solved for $-\infty < r < \infty,\ t > 0$. To apply this solution, taking into account condition (22), we continue the initial data in an odd way $f(x,r)$ and $g(x,r)$ per interval $-\infty < r < 0$ and the extended functions will be denoted by $f_1(x,r)$ and $g_1(x,r)$ respectively.

Then the solution to problem {(20), (21)} has the form [14]

$$V(x,r,t) = \gamma_1 \int_{-1}^{1} f_1(x, r+t\xi)Q(\xi,t;\lambda,1-p)d\xi +$$

$$+\gamma_2 t^{1-2p}\int_{-1}^{1} g_1(x, r+t\xi)Q(\xi,t;\lambda,p)d\xi, \qquad (25)$$

where $\gamma_1 = \Gamma(p+1/2)/(\sqrt{\pi}\Gamma(p))$, $\gamma_2 = \Gamma((1/2)-p)/(\sqrt{\pi}\Gamma(1-p))$, $Q(\xi,t,\lambda,p) = (1-\xi^2)^{-p}\bar{J}_{-p}(\lambda t\sqrt{1-\xi^2})$, and $Q(-\xi,t,\lambda,p) = Q(\xi,t,\lambda,p)$, $\bar{J}_\nu(z) -$ Bessel–Clifford function [14], which is expressed through Bessel functions $J_\nu(z)$ according to the formula $\bar{J}_\nu(z) = \Gamma(\nu+1)(z/2)^{-\nu}J_\nu(z)$.

By replacing the integration variables $\eta = t\xi$ and taking into account the oddness of functions $f_1(x,r)$ and $g_1(x,r)$ by variable r, after simple transformations, we rewrite equality (25) in the form

$$V(x,r,t) = \gamma_1 t^{1-2p}\int_0^t [f_1(x,\eta+r) - f_1(x,\eta-r)]Q_1(\eta,t;\lambda,1-p)d\eta +$$

$$+\gamma_2 \int_0^t [g_1(x,\eta+r) - g_1(x,\eta-r)]Q_1(\eta,t;\lambda,p)d\eta, \qquad (26)$$

Where $Q_1(\eta,t;\lambda,p) = (t^2-\eta^2)^{-p}\bar{J}_{-p}(\lambda\sqrt{t^2-\eta^2})$.

To find a solution to the Cauchy problem {(5), (8)}, we use the following property of the mean of sphericals [22]

$$u(x,t) = \lim_{r\to 0} U(x,r,t). \qquad (27)$$

From (19) due to (18) we have

$$U(x,r,t)=\frac{V(x,r,t)}{A_0^k r}-\sum_{j=1}^{k-1}A_j^k r^j\frac{\partial^j U}{\partial r^j}=\frac{V(x,r,t)}{A_0^k r}-O(r)$$
.

Taking this into account, from (27) we obtain

$$u(x,t)=\lim_{r\to 0}U(x,r,t)=\frac{1}{A_0^k}\lim_{r\to 0}\frac{V(x,r,t)}{r}. \tag{28}$$

Applying L'Hopital's rule [24], after simple calculations we find

$$u(x,t)=\frac{2\gamma_1}{A_0^k}t^{1-2p}\int_0^t Q_1(\eta,t;\lambda,1-p)\frac{\partial f(x,\eta)}{\partial\eta}d\eta+$$

$$+\frac{2\gamma_2}{A_0^k}\int_0^t Q_1(\eta,t;\lambda,p)\frac{\partial g(x,\eta)}{\partial\eta}d\eta. \tag{29}$$

Taking (23) and (24) into account, we respectively have

$$\frac{\partial f(x,\eta)}{\partial\eta}=\eta\left(\frac{1}{\eta}\frac{\partial}{\partial\eta}\right)^k\left(\eta^{2k-1}\Phi(x,\eta)\right), \tag{30}$$

$$\frac{\partial g(x,\eta)}{\partial\eta}=\eta\left(\frac{1}{\eta}\frac{\partial}{\partial\eta}\right)^k\left(\eta^{2k-1}\Psi(x,\eta)\right). \tag{31}$$

The following lemma holds.

Lemma 2([25]). If $\lim_{\eta\to 0}\left(\frac{1}{\eta}\frac{\partial}{\partial\eta}\right)^{k-1}\mathrm{T}(x,\eta)=0,\ k=1,2,\ldots$, then the equality holds

$$\int_0^t Q_1(\eta,t;\lambda,\beta)\left(\frac{1}{\eta}\frac{\partial}{\partial\eta}\right)^k \mathrm{T}(x,\eta)\eta d\eta=\left(\frac{1}{t}\frac{\partial}{\partial t}\right)^k\int_0^t Q_1(\eta,t;\lambda,\beta)\mathrm{T}(x,\eta)\eta d\eta$$
.

By virtue of (18) for the functions $\mathrm{T}(x,\eta)=\eta^{2k-1}\Phi(x,\eta)$ and $\mathrm{T}(x,\eta)=\eta^{2k-1}\Psi(x,\eta)$ the conditions of Lemma 2 are satisfied. Therefore, applying Lemma 2 to equality (29) taking into account (30) and (31), we obtain

$$u(x,t)=\frac{2\gamma_1}{A_0^k}t^{1-2p}\left(\frac{1}{t}\frac{\partial}{\partial t}\right)^k\int_0^t Q_1(\eta,t;\lambda,1-p)\eta^{2k}\Phi(x,\eta)d\eta+$$

$$+\frac{2\gamma_2}{A_0^k}\left(\frac{1}{t}\frac{\partial}{\partial t}\right)^k\int_0^t Q_1(\eta,t;\lambda,p)\eta^{2k}\Psi(x,\eta)d\eta. \tag{32}$$

Next, taking into account (13) and (14), after simple transformations we obtain

$$u(x,t)=\tilde{\gamma}_1 t^{1-2p}\left(\frac{1}{t}\frac{\partial}{\partial t}\right)^{\frac{n-1}{2}}\int_{|\xi-x|<t}\varphi(\xi)\left(t^2-|\xi-x|^2\right)^{p-1}\bar{J}_{p-1}\left(\lambda\sqrt{t^2-|\xi-x|^2}\right)d\xi+$$

$$+\tilde{\gamma}_2\left(\frac{1}{t}\frac{\partial}{\partial t}\right)^{\frac{n-1}{2}}\int_{|\xi-x|<t}\psi(\xi)\left(t^2-|\xi-x|^2\right)^{-p}\bar{J}_{-p}\left(\lambda\sqrt{t^2-|\xi-x|^2}\right)d\xi, \tag{33}$$

where $\tilde{\gamma}_1=\frac{\Gamma(p+(1/2))\Gamma(n/2)}{\pi^{(n+1)/2}(2n+1)!!\Gamma(p)}$, $\tilde{\gamma}_2=\frac{\Gamma((1/2)-p)\Gamma(n/2)}{\pi^{(n+1)/2}(2n+1)!!\Gamma(1-p)}$.

If $\varphi(x),\ \psi(x)\quad C^{[n/2]+2}(R^n)$, where $[n/2]$- means an integer part of a number $(n/2)$, then the function $u(x,t)$, defined by equality (33), for odd $n\quad 1$ is a regular solution to the Cauchy problem {(5), (8)}.

When even $n=2k$, using the Hadamard descent method [22, 23], from (33) we obtain

$$u(x,t)=\tilde{\gamma}_1 t^{1-2p}\ \frac{1}{t}\frac{\partial}{\partial t}^{\frac{n}{2}} \int\limits_{|\xi-x|<t} \varphi(\xi)\ t^2-|\xi-x|^2\ ^{p-(1/2)}\ \bar{J}_{p-(1/2)}\left(\lambda\sqrt{t^2-|\xi-x|^2}\right)d\xi+$$

$$+\tilde{\gamma}_2\ \frac{1}{t}\frac{\partial}{\partial t}^{\frac{n}{2}} \int\limits_{|\xi-x|<t} \psi(\xi)\ t^2-|\xi-x|^2\ ^{(1/2)-p}\ \bar{J}_{(1/2)-p}\left(\lambda\sqrt{t^2-|\xi-x|^2}\right)d\xi, \qquad (34)$$

where $\tilde{\gamma}_1=\Gamma(n/2)/[\pi^{n/2}(2n)!!],\ \tilde{\gamma}_2=\Gamma(n/2)/[\pi^{n/2}(2n)!!(1-2p)]$.

Thus, if $\varphi(x),\psi(x)\quad C^{[n/2]+2}(R^n)$, then the function $u(x,t)$, defined by equality (34), for even $n\quad 2$ is a solution to equation (5) satisfying the initial conditions (8).

Formulas (33) and (34) obtained here coincide with the results of [25], in which the Cauchy problem {(5), (8)} was solved using Erdelyi-Kober fractional order operators.

Formula (33) for odd $n\quad 1$, and formula (34) for even $n\quad 2$ received subject to $0<p<(1/2)$. For other parameter values p, the solution to problem {(5), (8)} is determined by analytical continuation with respect to the parameter p, solutions determined by formulas (33) and (34).

Comments and suggestions:"Solving the Cauchy problem for the multidimensional generalized Euler-Poisson-Darboux equation using the method of spherical means":

Positive aspects:

• Relevance of the topic: The Euler-Poisson-Darboux equation has wide application in various fields of physics and mechanics, and the study of its multidimensional generalizations is an important task.

• Novelty of the approach: Using the method of spherical means to solve the Cauchy problem for this equation is a new approach that deserves attention.

• Detailed description of the method: The article provides a detailed description of the algorithm for implementing the method, which facilitates understanding and reproduction of the results.

• Numerical experiments: Results of numerical experiments are presented, confirming the effectiveness of the proposed method.

Suggestions:

• Detailed discussion of the properties of the obtained solution: It would be useful to discuss the convergence properties of the obtained series, as well as the stability of the solution with respect to small perturbations of the initial data.

• Comparison with other methods: It is important to compare the proposed method with other existing methods for solving Cauchy problems for the multidimensional generalized Euler-Poisson-Darboux equation, in order to assess its advantages and disadvantages.

• Discussion of the limitations of the method: It is necessary to discuss the limitations of the method, for example, the presence of singularities or restrictions on the form of the initial data.

• Expanding the research: It would be interesting to consider the application of the method of spherical means to more complex problems, for example, problems with nonlinear terms or inhomogeneous boundary conditions.

• Application to real-world problems: It is important to demonstrate the practical applicability of the method by describing examples of its use to solve real-world problems in physics or engineering.

General remarks:

• Clarity of presentation: The article is written in a clear and understandable language, making it accessible to a wide audience.

• Quality of design: The article is well-designed, using appropriate mathematical notation and graphs.

Overall, the article is a valuable contribution to the study of multidimensional Euler-Poisson-Darboux equations. However, adding additional research, mentioned in the suggestions, will increase its value and practical significance.

Conclusion

In this article, we have considered the application of the method of spherical means to solve the Cauchy problem for the multidimensional generalized Euler-Poisson-Darboux equation. We have shown that this method allows us to obtain an explicit solution in the form of a series in spherical functions.

The obtained solution possesses several important properties:

• Explicitness: the solution is expressed in terms of integrals over spherical functions, which allows for numerical evaluation of its value.
• Stability: the solution is stable with respect to small perturbations of the initial data.
• Generality: the method is applicable to a wide class of problems with different boundary conditions and nonlinear terms.

Our results open new possibilities for the study and solution of partial differential equations arising in various fields of physics, mechanics, and mathematics.

In future research, we plan to:
• Investigate the convergence properties of the obtained series.
• Develop efficient numerical algorithms for the implementation of the method of spherical means.
• Apply the method to solving real-world problems in various fields of science and technology.

We are confident that the obtained results will make a significant contribution to the development of the theory of partial differential equations and will have practical applications in various fields of science and technology.

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