



# DETERMINING ELASTIC CROSS-SECTIONS IN POSITRON-HELIUM ION COLLISIONS

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## Abstract

*This study investigates the elastic cross-sections of positron scattering from helium ions, an important process in understanding the fundamental interactions in atomic and plasma physics. Using advanced computational techniques and quantum mechanical models, we calculate the differential and total elastic cross-sections for positron collisions with singly and doubly ionized helium ions over a range of incident energies. The results are analyzed to reveal the underlying scattering mechanisms and to compare with existing experimental and theoretical data. Our findings provide new insights into the role of electron correlation and positronium formation in these collisions, contributing to the broader knowledge of positron interactions with complex atomic systems. This study also discusses the implications of these cross-sections for applications in fields such as astrophysics, plasma diagnostics, and radiation therapy.*

## Keywords

*Elastic cross-sections, positron scattering, helium ions, quantum mechanics, positron-atom collisions, differential cross-sections, total cross-sections, electron correlation, positronium formation, atomic physics, computational techniques, plasma diagnostics, radiation therapy, astrophysics.*

## INTRODUCTION

The study of elastic cross-sections in positron-helium ion collisions is essential for advancing our understanding of fundamental particle interactions in atomic and plasma physics. Positrons, the antimatter counterparts of electrons, have unique interactions with matter, offering valuable insights into electron correlation effects and atomic structure. When a positron collides with a helium ion, the nature of the interaction can reveal detailed information about the scattering process, including the distribution of cross-sections and the dynamics of the interaction. The elastic scattering of positrons is particularly intriguing due to the potential formation of positronium—a quasi-stable bound state of an electron and positron—which adds complexity to the theoretical models used to describe these collisions.

Helium ions, due to their relatively simple electronic structure and the presence of multiple ionization states (singly and doubly ionized), provide an ideal system for studying positron interactions. The investigation of

positron scattering from helium ions not only enhances our fundamental knowledge but also has practical implications in various fields. For example, in plasma physics, understanding these interactions is crucial for developing accurate models of plasma behavior in both laboratory and astrophysical environments. In radiation therapy, positron scattering data can improve the accuracy of dose calculations, particularly in treatments involving positron emission tomography (PET).

Theoretical studies of positron-helium ion collisions typically involve quantum mechanical approaches such as the close-coupling method, the Born approximation, or complex potential models, all of which aim to accurately describe the interaction potential and resulting scattering phenomena. Computational advancements have enabled more precise calculations of elastic cross-sections, incorporating factors like polarization effects and exchange interactions that are pivotal at low to intermediate energies. Despite these advancements, discrepancies remain between theoretical predictions and experimental measurements, particularly at higher energies where inelastic processes can complicate the scattering dynamics.

This research aims to provide a comprehensive analysis of elastic cross-sections for positron scattering from helium ions, focusing on the differential and total cross-sections over a range of incident positron energies. By employing state-of-the-art computational methods and comparing results with existing experimental data, this study seeks to resolve some of the outstanding questions in the field and offer new insights into the role of electron correlation and positronium formation. Ultimately, understanding these scattering processes can contribute significantly to the broader scientific knowledge of positron interactions with atomic and ionic systems, supporting developments in both theoretical and applied physics.

## METHOD

To determine the elastic cross-sections for positron scattering from helium ions, we employed a combination of theoretical and computational techniques designed to accurately model the quantum mechanical interactions between positrons and both singly ( $\text{He}^+$ ) and doubly ( $\text{He}^{++}$ ) ionized helium ions. Our approach is based on solving the Schrödinger equation for the scattering system, taking into account the Coulomb potential and other relevant interaction potentials. The methodology is divided into several key steps, including the formulation of the interaction potentials, the application of computational techniques for solving the scattering equations, and the calculation of both differential and total cross-sections.

The first step in our methodology involved defining the potential models for the positron-helium ion system. For the  $\text{He}^+$  ion, the interaction potential consists of the Coulomb attraction between the positron and the nucleus, modulated by the screening effect of the remaining bound electron. To model this system accurately, we used a modified Coulomb potential that accounts for the shielding effect of the electron cloud. Additionally, polarization effects were considered, particularly at low incident energies where the positron might induce dipole moments in the helium ion.

For the  $\text{He}^{++}$  ion, the interaction potential is purely Coulombic due to the absence of any electrons, leading to a straightforward repulsive potential between the positron and the doubly charged nucleus. In this case, the potential is simpler, but the resulting scattering equations require careful treatment of the singularities associated with the Coulomb interaction at small impact parameters.

With the potentials defined, we proceeded to solve the scattering equations using a combination of the

close-coupling method and the Born approximation. The close-coupling method allows for a detailed description of the interaction by expanding the wave function of the scattering system in terms of a set of coupled differential equations, which are then solved numerically. This method is particularly useful at low to intermediate energies where the positron can strongly interact with the target ion, potentially forming intermediate states such as positronium.

The Born approximation, on the other hand, provides a perturbative approach suitable for high-energy scattering scenarios where the interaction between the positron and the ion is weaker, allowing for simplifications in the scattering potential. In our study, we applied the first Born approximation for energies above 500 eV, where the interaction potential can be treated as a small perturbation to the free-particle motion of the positron.

To solve these equations numerically, we implemented a combination of finite-difference methods and matrix diagonalization techniques. The differential cross-sections were obtained by solving the angular-dependent scattering equations, while the total cross-sections were computed by integrating the differential cross-sections over all solid angles. These computations were performed using a high-performance computing cluster to ensure precision and to handle the computational complexity of the coupled equations. The differential cross-section,  $d\sigma/d\Omega$ , provides the probability of scattering into a specific solid angle and is crucial for understanding the angular distribution of scattered positrons. We calculated the differential cross-sections for both He<sup>+</sup> and He<sup>++</sup> ions across a range of incident energies from 10 eV to 1 keV. For each energy, we analyzed the angular dependence of the cross-sections to identify features such as forward scattering peaks, which are indicative of long-range Coulomb interactions, and any anomalies that may suggest the presence of complex interaction dynamics.

The total elastic cross-section,  $\sigma_{\text{total}}$ , was determined by integrating the differential cross-sections over all scattering angles. This quantity represents the overall likelihood of elastic scattering occurring during a positron-ion collision and is essential for comparing our theoretical predictions with experimental data. Our computations of total cross-sections were validated against known benchmarks for positron-electron systems to ensure the reliability of our numerical methods.

To ensure the accuracy of our results, we compared our theoretical cross-sections with available experimental data and other theoretical models reported in the literature. This comparison was conducted over a range of energies where data were available, particularly focusing on the low to intermediate energy regions where both theory and experiment provide comprehensive datasets. Any discrepancies observed were further analyzed to understand potential sources of error, such as approximations in the interaction potential or limitations of the computational methods used.

Additionally, we performed convergence tests to ensure that our numerical results were independent of the grid size and other computational parameters. These tests confirmed that the results were stable and accurate across the energy range considered, with uncertainties primarily arising from physical approximations rather than numerical errors.

Throughout the calculations, special attention was given to capturing the effects of electron correlation, exchange interactions, and polarization. In the low-energy regime, where these effects are most pronounced, we employed a more sophisticated multi-channel close-coupling approach to account for the potential formation of temporary positronium states and to model exchange effects accurately. At higher energies,

where relativistic effects could become significant, we considered higher-order corrections to the scattering amplitude to improve the fidelity of our predictions.

By incorporating these diverse theoretical and computational techniques, our study provides a comprehensive framework for understanding the elastic scattering of positrons from helium ions. The results obtained offer valuable insights into the complex dynamics governing these interactions and lay the groundwork for future investigations into more complex atomic and molecular systems.

## RESULTS

The results of this study provide a detailed analysis of the elastic cross-sections for positron scattering from helium ions, focusing on both singly ( $\text{He}^+$ ) and doubly ( $\text{He}^{++}$ ) ionized states. Using advanced computational techniques, including the close-coupling method and the Born approximation, we calculated differential and total elastic cross-sections over a range of positron incident energies, spanning from low (10 eV) to high (1 keV) energies. Our findings reveal distinct variations in the scattering behavior depending on the ionization state of helium and the energy of the incident positrons.

For  $\text{He}^+$  ions, the differential cross-sections exhibit pronounced peaks at specific scattering angles, corresponding to the influence of the Coulomb attraction between the positron and the remaining electron. At lower energies, the cross-sections show strong forward scattering, indicating a significant contribution from long-range Coulomb interactions. As the energy increases, the scattering pattern becomes more isotropic, with a noticeable reduction in cross-section values due to the decrease in interaction time between the positron and the ion. The total cross-section for  $\text{He}^+$  shows a gradual decline with increasing energy, consistent with the decreasing probability of elastic scattering events at higher velocities.

In contrast, for  $\text{He}^{++}$  ions, the absence of an electron leads to a purely repulsive Coulomb interaction with the positron, resulting in a different scattering profile. The differential cross-sections for  $\text{He}^{++}$  are characterized by a lack of forward-peaking behavior, and instead, exhibit a more uniform distribution across scattering angles, especially at higher incident energies. The total elastic cross-section for  $\text{He}^{++}$  is consistently lower than that of  $\text{He}^+$ , reflecting the reduced probability of positron capture and elastic scattering without the influence of an electron cloud. Interestingly, at very low incident energies, there is a slight increase in the cross-section, suggesting a potential influence of polarization effects, which diminish rapidly as energy increases.

Comparing our theoretical results with available experimental data, we find good agreement in the low to intermediate energy ranges (10-200 eV) for both  $\text{He}^+$  and  $\text{He}^{++}$  ions. However, discrepancies arise at higher energies (above 500 eV), where experimental cross-sections tend to be lower than our theoretical predictions. This divergence may be attributed to the onset of inelastic processes, such as ionization or positronium formation, which are not fully accounted for in our elastic scattering models. Additionally, the role of higher-order corrections, such as relativistic effects and exchange interactions, becomes more pronounced at these energies, suggesting avenues for further refinement of our theoretical framework.

Overall, the results highlight the complex nature of positron-helium ion collisions, with distinct scattering characteristics depending on the ionization state and energy of the interacting particles. Our study not only advances the understanding of these fundamental interactions but also provides a valuable reference for

future experimental and theoretical investigations in atomic and plasma physics. The insights gained from this research may have broader implications for applications in materials science, radiation therapy, and astrophysical modeling, where accurate knowledge of positron scattering cross-sections is essential.

## DISCUSSION

The findings from our study on elastic cross-sections in positron-helium ion collisions provide significant insights into the interaction dynamics between positrons and ionized helium atoms. The results highlight several key features that distinguish the scattering behaviors of singly ( $\text{He}^+$ ) and doubly ( $\text{He}^{++}$ ) ionized helium ions, particularly in relation to the energy of the incident positrons. For  $\text{He}^+$  ions, the observed differential cross-section patterns—characterized by strong forward scattering at low energies—underscore the influence of long-range Coulomb attraction and the screening effect provided by the remaining electron. This behavior suggests that at low incident energies, the positron remains in the vicinity of the helium ion long enough for substantial interaction, including possible temporary positronium formation or strong polarization effects. As the incident positron energy increases, the cross-sections become more isotropic, reflecting a decrease in interaction time and reduced influence of polarization effects. The gradual decrease in the total cross-section with increasing energy aligns well with theoretical expectations of reduced scattering probabilities at higher velocities.

In contrast, the scattering from  $\text{He}^{++}$  ions exhibits a distinctively different pattern due to the purely repulsive nature of the Coulomb interaction between the positively charged ion and the positron. The lack of forward scattering peaks and the relatively uniform angular distribution at higher energies indicate that the absence of electron shielding leads to simpler, more straightforward scattering dynamics dominated by the repulsive Coulomb force. The lower total elastic cross-section observed for  $\text{He}^{++}$  ions compared to  $\text{He}^+$  ions across all energy ranges can be attributed to the lack of attractive interactions that would otherwise increase the scattering cross-section.

Our theoretical results show good agreement with experimental data in the low to intermediate energy range (10-200 eV), which validates our computational models and the interaction potentials used. However, discrepancies at higher energies (above 500 eV) suggest the need to consider additional factors such as inelastic processes, including positronium formation, ionization, and excitation of the helium ion. These processes become increasingly probable at higher energies, which may lead to a reduction in observed elastic cross-sections as some positrons undergo inelastic collisions that are not accounted for in our purely elastic scattering model.

Moreover, the study highlights the importance of incorporating higher-order corrections and exchange effects to improve theoretical predictions, especially at energies where relativistic effects may become significant. The observed discrepancies also indicate the potential role of multi-channel scattering processes, where the positron can induce transitions to excited states of the helium ion, thereby modifying the elastic scattering cross-sections.

Future work should focus on extending the model to include these inelastic channels and exploring the impact of relativistic effects more comprehensively. Additionally, more precise experimental measurements at high energies could help refine the theoretical models and clarify the underlying mechanisms of positron-

ion interactions. Overall, this study advances our understanding of positron scattering dynamics and provides a robust framework for analyzing similar interactions in more complex atomic and molecular systems. These insights are crucial for applications in fields such as astrophysics, where positron collisions with ions are common, and in medical physics, where understanding positron interactions can improve imaging and treatment modalities.

## CONCLUSION

This study provides a comprehensive analysis of elastic cross-sections for positron scattering from helium ions, focusing on both singly ( $\text{He}^+$ ) and doubly ( $\text{He}^{++}$ ) ionized states. By employing advanced computational techniques, including the close-coupling method and the Born approximation, we have successfully calculated differential and total cross-sections over a wide range of incident positron energies. The results highlight distinct scattering behaviors depending on the ionization state and energy, with  $\text{He}^+$  ions showing significant forward scattering at low energies due to the combined effects of Coulomb attraction and electron shielding, while  $\text{He}^{++}$  ions exhibit more uniform angular distributions driven by purely repulsive Coulomb forces.

Our findings align well with experimental data at low to intermediate energies, validating the robustness of our theoretical models. However, the observed discrepancies at higher energies suggest the presence of inelastic processes and underscore the need for further refinement of the theoretical framework, including the incorporation of positronium formation, ionization, and other inelastic channels. Additionally, our study points to the importance of considering higher-order effects, such as relativistic and exchange interactions, particularly at higher energies where these factors could significantly impact scattering outcomes.

Overall, this research advances our understanding of positron-helium ion collisions and provides a foundation for future studies aimed at more accurately modeling these interactions. The insights gained from this work are not only relevant to atomic and plasma physics but also have broader implications for applications in astrophysics, materials science, and medical physics, where knowledge of positron interactions plays a critical role. Moving forward, further experimental and theoretical investigations are needed to explore the complex interplay of factors influencing positron scattering and to refine our models to achieve greater accuracy and predictive power.

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